

Assume that the calorimeter is being used to select the desired energy photon. Assume the positrons/pulse times radiator thickness are adjusted to give $n\gamma_{avg}$ "full spectrum photons" per pulse.

$E_{min} := 20$ [Mev] minimum energy converable gamma

$$\text{poisson}(n, n_{avg}) := \frac{e^{-n_{avg}} \cdot n_{avg}^n}{n!}$$

$fracwidth := .5$ fractional full width of the gamma's energy bin

Calculate the probability per pulse that there is exactly one brems photon in the calorimeter bin energy range. This is the poisson of 1 given the avrg number of photons per pulse in the bin.

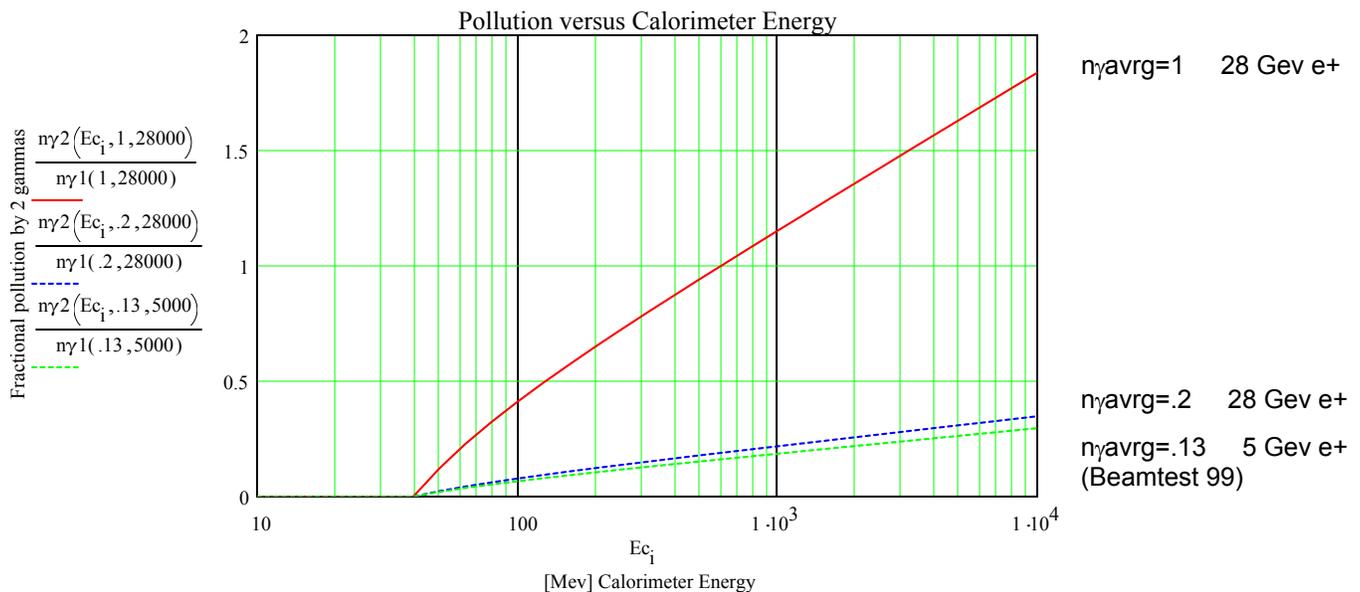
$$n\gamma_1(n\gamma_{avg}, E_{beam}) := \text{poisson}\left[1, n\gamma_{avg} \cdot \left(\frac{1}{\ln\left(\frac{E_{beam}}{E_{min}}\right)} \cdot fracwidth\right)\right]$$

Calculate the probability per pulse that there are exactly two photons that sum up to be in calorimeter energy bin. Since we select on calorimeter energy, we can not tell that there are really two photons of undesired lower energies.

$$n\gamma_2(E_{cal}, n\gamma_{avg}, E_{beam}) := \left(\frac{n\gamma_{avg}}{\ln\left(\frac{E_{beam}}{E_{min}}\right)}\right)^2 \cdot \int_{E_{min}}^{\text{if}((E_{cal} - E_{min}) < E_{min}, E_{min}, E_{cal} - E_{min})} \frac{1}{E \cdot (E_{cal} - E)} dE \cdot (fracwidth \cdot E_{cal})$$

Not all of these two photon events are bad, though we are assuming they are for simplicity. One photon may be within fracwidth. Then there is only a 50% chance that the lower energy photon is the one to convert and pollute the point spread function measurement. If both photons are outside fracwidth, then there is a 100% chance that the conversion is pollution.

$$i := 0 .. 30 \quad E_{c_i} := 10^{\frac{i}{10} + 1}$$



Comparison to Beam Test 99 Data
(Eduardo's analysis)

Typical Beamtest 1999: k := 0..6

npos := .9 run_k := pulses_k :=

trad := .027

333	114941
334	108754
336	106855
337	109472
919	129461
920	81471
930	70254

$$\text{totpulses} := \sum_k \text{pulses}_k \quad \text{totpulses} = 721208$$

$$n\gamma_{\text{avg}} := \text{npos} \cdot \text{trad} \cdot \ln\left(\frac{5000}{E_{\text{min}}}\right)$$

$n\gamma_{\text{avg}} = 0.1342$ Avg number of full spectra gammas per pulse for Beamtest 99.

$$n\gamma_{\text{avg_Ebin}} := \text{npos} \cdot \text{trad} \cdot \ln\left(\frac{3200}{1600}\right)$$

$n\gamma_{\text{avg_Ebin}} = 0.0168$ Avg number of gammas/pulse in Ebin

$$\text{ntot_bin} := \text{totpulses} \cdot \text{poisson}(1, n\gamma_{\text{avg_Ebin}})$$

$\text{ntot_bin} = 11945$ Total number of beam gammas in the energy bin

$$\text{converter} := 10 \cdot (.036 + .013) + 0 \cdot (.28 + .013)$$

$\text{converter} = 0.49$ Tracker radiation lengths. There are 11 thin converter layers. Eduardo did not use the top most one, and also did not use the 3 super GLAST converters.

Number of gammas converted

$$n\gamma_{\text{convrtd}} := \text{totpulses} \cdot \text{poisson}\left[1, n\gamma_{\text{avg_Ebin}} \cdot \left(1 - e^{\frac{-7}{9} \cdot \text{converter}}\right)\right]$$

$n\gamma_{\text{convrtd}} = 3829$

Number of gamma candidates found.

$$n\gamma_{\text{found}} := 1377 + 1095 + 1045$$

$n\gamma_{\text{found}} = 3517$

Reconstruction efficiency

$$\text{recon_effic} := \frac{n\gamma_{\text{found}}}{n\gamma_{\text{convrtd}}}$$

$\text{recon_effic} = 0.92$

This is a reasonable reconstruction efficiency. Conclude that there is some agreement between the calculated and measured number of photons.

Radiative correction also gives 2 photon events:

For Beamtest 2004 choose $n_{\text{pos}} \cdot \text{trad}$ to give an average of .2 total photons/pulse.

$$n_{\text{pos}} := 1$$

$$\text{trad} := .0276$$

$$n\gamma_{\text{avg}} := n_{\text{pos}} \cdot \text{trad} \cdot \ln\left(\frac{28000}{20}\right) \quad n\gamma_{\text{avg}} = 0.1999$$

Sometimes the positron will radiate twice (in different places going through the radiator) in trad. Then the spectrum of the two photons is:

$$\left(\text{trad} \cdot \frac{dE1}{E1}\right) \times \left(\text{trad} \cdot \frac{dE2}{E2}\right)$$

Sometimes the positron will radiate twice while it interacts with just one nucleus (ie: a radiative correction on the electron line). Then the spectrum of the two photons is approx:

$$\left(\text{trad_equiv} \cdot \frac{dE1}{E1}\right) \times \left(\text{trad} \cdot \frac{dE2}{E2}\right)$$

$$\alpha := \frac{1}{137}$$

$$\text{trad_equiv} := 2 \cdot \frac{\alpha}{\pi}$$

$$\text{trad_equiv} = 0.0046$$

So, we get a few more 2 photon events than expected, but they have the same differential spectrum in E1, E2. The fractional increase in the number of 2 photon events is:

$$\frac{(\text{trad_equiv} + \text{trad}) \cdot \text{trad}}{\text{trad} \cdot \text{trad}} = 1.1684$$