

Consider a silicon tracker telescope at the input and output of the magnet so that the in and out directions are determined. The beam first enters xSi det 1. Assume xSi dets 1 and 2 are dA apart and also xSi dets 3 and 4 are dA apart. The distance from det 2 to det 3 is dB. The bend angle is the difference between the directions measured by the two pairs.

Choose dA so that the uncertainty in angle caused by the strip pitch is less than the uncertainty in angle caused by multiple scattering in det2 + air + det3 (for the highest energy positron we might use = 1 Gev). dB should not be too small for a realistic magnet. θ_{inside} should be large enough so that there is space for shielding to absorb the deflected electron without bothering the radiated photon that is continuing down the beamline.

1) First make a configuration with the lowest energy positron beam we could possibly get down the A line (1 Gev is optimistic)

$$\sigma_{\text{si}} := \frac{242 \cdot 10^{-6}}{\sqrt{12}}$$

[m] Si pitch

$$\text{width} := .096$$

[m] Width of det 4

$$dA := .20$$

[m] Distance from det 1 to 2 = det 3 to 4

$$dB := 1.0$$

[m] Distance from det 2 to 3

$$E_{\text{gmax}} := 500$$

[MeV] Max energy photon (beam outer edge of det 4)
[MeV] Positron beam energy

$$E_{\text{beam}} := 1000$$

Frac energy spread of beam (slits wide open for rate)

$$\sigma_{\text{frac}} := .02$$

$$X0 := \frac{1.400 \cdot 10^{-6}}{9.36 \cdot 10^{-2}} + \frac{dA + dB + dA}{304.20} + \frac{1.400 \cdot 10^{-6}}{9.36 \cdot 10^{-2}} \quad X0 = 0.013$$

Number of rad lengths (xydet 2 + air + xydet 3)
Xair=304.20 m , XHe=5299 m

$$BL := \frac{\text{width}}{dA + \frac{dB}{2}} \cdot \frac{1}{.03} \cdot \frac{1}{10^3} \cdot \left(\frac{1}{E_{\text{beam}} - E_{\text{gmax}}} - \frac{1}{E_{\text{beam}}} \right)^{-1} \quad BL = 4.571$$

[kGauss-m] Magnet bending strength to put the beam on the inside edge of det 4 and the E_{gmax} radiated beam on the outside edge of det 4.

$$\theta_{\text{inside}} := \frac{.03 \cdot 10^3 \cdot BL}{E_{\text{beam}}}$$

$$\theta_{\text{inside}} \cdot \left(\frac{180}{\pi} \right) = 7.858$$

[deg] Angle from beamline to inner edge of det 4

$$\theta_{\text{outside}} := \frac{.03 \cdot 10^3 \cdot BL}{E_{\text{beam}} - E_{\text{gmax}}}$$

$$\theta_{\text{outside}} \cdot \left(\frac{180}{\pi} \right) = 15.715$$

[deg] Angle from beamline to outer edge of det 4

$$d_{\text{inside4}} := \theta_{\text{inside}} \cdot \left(dA + \frac{dB}{2} \right)$$

$$d_{\text{inside4}} = 0.096$$

[m] Distance from beamline to outer edge of det 4

$$d_{\text{outside4}} := \theta_{\text{outside}} \cdot \left(dA + \frac{dB}{2} \right)$$

$$d_{\text{outside4}} = 0.192$$

[radians] Bend angle error due to silicon strip pitch

$$\sigma_{\theta_{\text{si}}} := \sqrt{2 \cdot \left(\frac{\sigma_{\text{si}}}{dA} \right)^2 + 2 \cdot \left(\frac{\sigma_{\text{si}}}{dA} \right)^2}$$

$$\sigma_{\theta_{\text{si}}} = 6.986 \cdot 10^{-4}$$

[radians] Bend angle error due to multiple scattering at Ebeam.

$$\sigma_{\text{ms}} := \frac{13.6}{E_{\text{beam}}} \cdot \sqrt{X0}$$

$$\sigma_{\text{ms}} = 1.56 \cdot 10^{-3}$$

$$\sigma_{\text{gam}}(E_{\text{beam}}, E_{\text{gmax}}, \sigma_{\text{frac}}, E_{\text{gam}}, dA, dB) :=$$

$$X0 \leftarrow \frac{1 \cdot 400 \cdot 10^{-6}}{9.36 \cdot 10^{-2}} + \frac{dA + dB + dA}{304.20} + \frac{1 \cdot 400 \cdot 10^{-6}}{9.36 \cdot 10^{-2}}$$

$$\sigma_{\theta \text{si}} \leftarrow \sqrt{2 \cdot \left(\frac{\sigma_{\text{si}}}{dA}\right)^2 + 2 \cdot \left(\frac{\sigma_{\text{si}}}{dA}\right)^2}$$

$$BL \leftarrow \frac{\text{width}}{dA + \frac{dB}{2}} \cdot \frac{1}{.03} \cdot \frac{1}{10^3} \cdot \left(\frac{1}{E_{\text{beam}} - E_{\text{gmax}}} - \frac{1}{E_{\text{beam}}}\right)^{-1}$$

$$\theta \leftarrow \frac{.03 \cdot 10^3 \cdot BL}{E_{\text{beam}} - E_{\text{gam}}}$$

$$\sigma_{\theta \text{ms}} \leftarrow \frac{13.6}{E_{\text{beam}} - E_{\text{gam}}} \cdot \sqrt{X0}$$

$$\sigma_{E_{\text{out}}} \leftarrow \frac{E_{\text{beam}} - E_{\text{gam}}}{\theta} \cdot \sqrt{(\sigma_{\theta \text{si}})^2 + (\sigma_{\theta \text{ms}})^2}$$

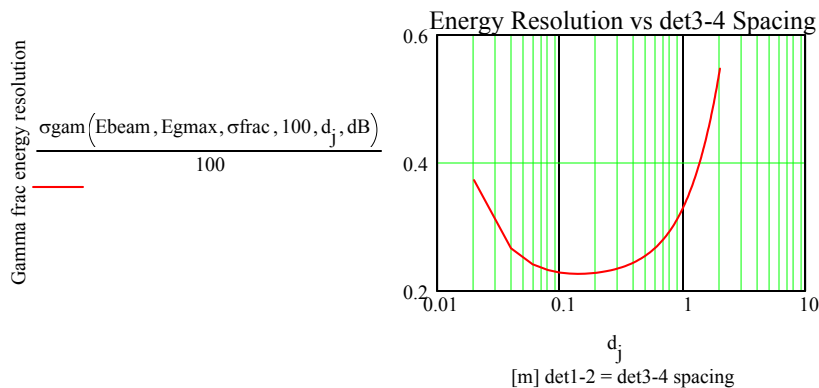
$$\sigma_{E_{\text{beam}}} \leftarrow \sigma_{\text{frac}} \cdot E_{\text{beam}}$$

$$\sigma_{\text{gam}} \leftarrow \sqrt{(\sigma_{E_{\text{beam}}})^2 + (\sigma_{E_{\text{out}}})^2}$$

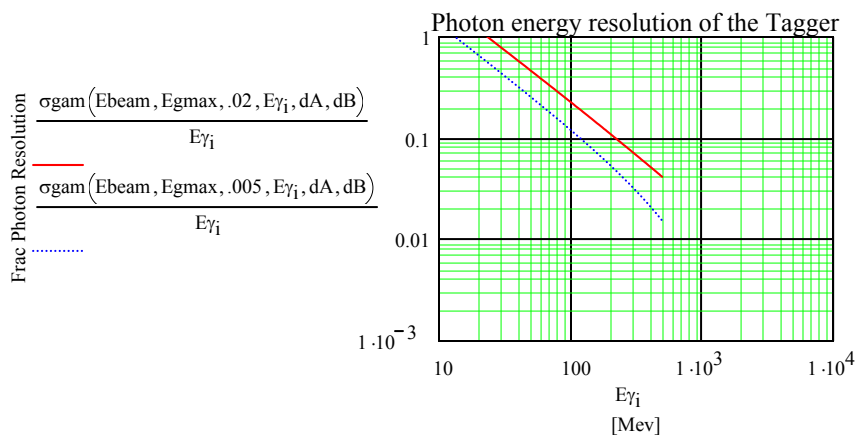
$$\sigma_{\text{gam}}$$

$$\text{nbins} := 100 \quad i := 0 .. \text{nbins} - 1 \quad E_{\gamma_i} := E_{\text{gmax}} \cdot \frac{(i+1)}{\text{nbins}} \quad j := 0 .. 100 \quad d_j := (j+1) \cdot .02$$

Find the telescope det1-det2 = det3-det4 = dA spacing that minimizes the gamma energy resolution at 100 Mev by varying dA.



dA = 0.2 [m] is pretty close to the minimum



Photon resolution of the tagger for A-line momentum slit widths of 2% and .5%. The .5% is clearly better, but the positron rate may be too low.

2) For a second configuration, choose a higher beam energy that will give photons that will overlap with the lowest energy positrons (1 Gev) that we expect to use for energy calibration.

By making (Ebeam, Egmax) a multiple (=3) of the previous values and leaving dA and dB the same, all we have to do is turn up BL to change configurations.

$$E_{gmax} := 1500$$

$$E_{beam} := 3000$$

$$\sigma_{frac} := .01$$

$$X0 := \frac{1.400 \cdot 10^{-6}}{9.36 \cdot 10^{-2}} + \frac{dA + dB + dA}{304.20} + \frac{1.400 \cdot 10^{-6}}{9.36 \cdot 10^{-2}} \quad X0 = 0.013$$

$$BL := \frac{width}{dA + \frac{dB}{2}} \cdot \frac{1}{.03} \cdot \frac{1}{10^3} \cdot \left(\frac{1}{E_{beam} - E_{gmax}} - \frac{1}{E_{beam}} \right)^{-1} \quad BL = 13.714$$

$$\theta_{inside} := \frac{.03 \cdot 10^3 \cdot BL}{E_{beam}} \quad \theta_{inside} \cdot \left(\frac{180}{\pi} \right) = 7.858$$

$$\theta_{outside} := \frac{.03 \cdot 10^3 \cdot BL}{E_{beam} - E_{gmax}} \quad \theta_{outside} \cdot \left(\frac{180}{\pi} \right) = 15.715$$

$$d_{inside4} := \theta_{inside} \cdot \left(dA + \frac{dB}{2} \right) \quad d_{inside4} = 0.096$$

$$d_{outside4} := \theta_{outside} \cdot \left(dA + \frac{dB}{2} \right) \quad d_{outside4} = 0.192$$

$$\sigma_{\theta si} := \sqrt{2 \cdot \left(\frac{\sigma_{si}}{dA} \right)^2 + 2 \cdot \left(\frac{\sigma_{si}}{dA} \right)^2} \quad \sigma_{\theta si} = 6.986 \cdot 10^{-4}$$

$$\sigma_{ms} := \frac{13.6}{E_{beam}} \cdot \sqrt{X0} \quad \sigma_{ms} = 5.198 \cdot 10^{-4}$$

[Mev] Max energy photon (beam outer edge of det 4)

[Mev] Positron beam energy

Frac energy spread of beam (slits 1/2 wide open for rate)

Number of rad lengths (xydet 2 + air + xydet 3)
Xair=304.20 m , XHe=5299 m

[kGauss-m] Magnet bending strength to put the beam on the inside edge of det 4 and the Egmax radiated beam on the outside edge of det 4.

[deg] Angle from beamline to inner edge of det 4

[deg] Angle from beamline to outer edge of det 4

[m] Distance from beamline to inner edge of det 4

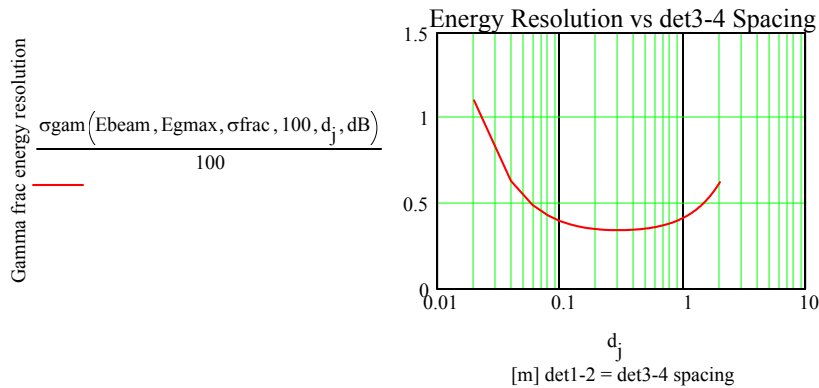
[m] Distance from beamline to outer edge of det 4

[radians] Bend angle error due to silicon strip pitch

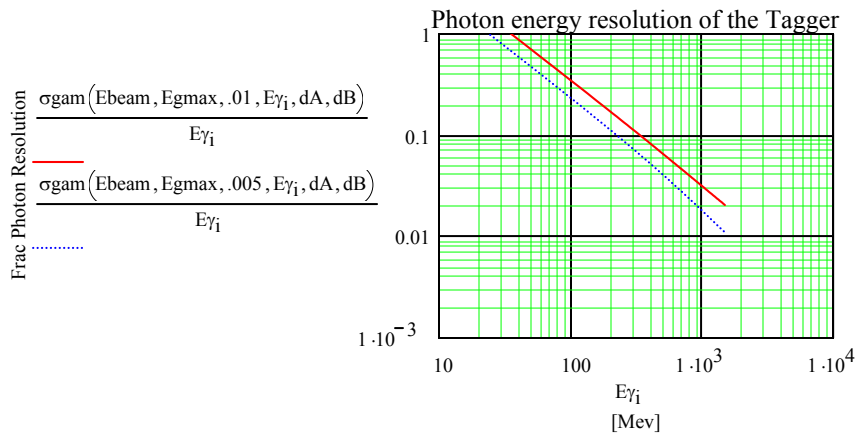
[radians] Bend angle error due to multiple scattering at Ebeam

$$n_{bins} := 100 \quad i := 0 .. n_{bins} - 1 \quad E_{\gamma_i} := E_{gmax} \cdot \frac{(i + 1)}{n_{bins}} \quad j := 0 .. 100 \quad d_j := (j + 1) \cdot 0.02$$

Find the telescope $det1-det2 = det3-det4 = dA$ spacing that minimizes the gamma energy resolution at 100 Mev by varying dA .



$dA = 0.2$ [m] is pretty close to the minimum



Photon resolution of the tagger for A-line momentum slit widths of 1% and .5%. The .5% is clearly better, but the positron rate may be too low.

3) For a third configuration, find an increased magnet setting that will tag a reasonable range of photon energies for our bulk of 28 GeV bremsstrahlung running. It is not in the Beam Test Plan to run this way.

$$dB := 5.0$$

[m] Distance from det 2 to det 3

$$Egmax := 14000$$

[MeV] Max energy photon (beam outer edge of det 4)

$$Ebeam := 28000$$

[MeV] Positron beam energy

$$\sigmafrac := .005$$

Frac energy spread of beam (slits 1/4 open for rate)

$$X0 := \frac{1 \cdot 400 \cdot 10^{-6}}{9.36 \cdot 10^{-2}} + \frac{dA + dB + dA}{304.20} + \frac{1 \cdot 400 \cdot 10^{-6}}{9.36 \cdot 10^{-2}} \quad X0 = 0.026$$

Number of rad lengths (xydet 2 + air + xydet 3)
Xair=304.20 m , XHe=5299 m

$$BL := \frac{\text{width}}{dA + \frac{dB}{2}} \cdot \frac{1}{.03} \cdot \frac{1}{10^3} \cdot \left(\frac{1}{Ebeam - Egmax} - \frac{1}{Ebeam} \right)^{-1} \quad BL = 33.185$$

[kGauss-m] Magnet bending strength to put the beam on the inside edge of det 4 and the Egmax radiated beam on the outside edge of det 4.

$$\theta_{inside} := \frac{.03 \cdot 10^3 \cdot BL}{Ebeam} \quad \theta_{inside} \cdot \left(\frac{180}{\pi} \right) = 2.037$$

[deg] Angle from beamline to inner edge of det 4

$$\theta_{outside} := \frac{.03 \cdot 10^3 \cdot BL}{Ebeam - Egmax} \quad \theta_{outside} \cdot \left(\frac{180}{\pi} \right) = 4.074$$

[deg] Angle from beamline to outer edge of det 4

$$d_{inside4} := \theta_{inside} \cdot \left(dA + \frac{dB}{2} \right) \quad d_{inside4} = 0.096$$

[m] Distance from beamline to inner edge of det 4

[m] Distance from beamline to outer edge of det 4

$$d_{outside4} := \theta_{outside} \cdot \left(dA + \frac{dB}{2} \right) \quad d_{outside4} = 0.192$$

[radians] Bend angle error due to silicon strip pitch

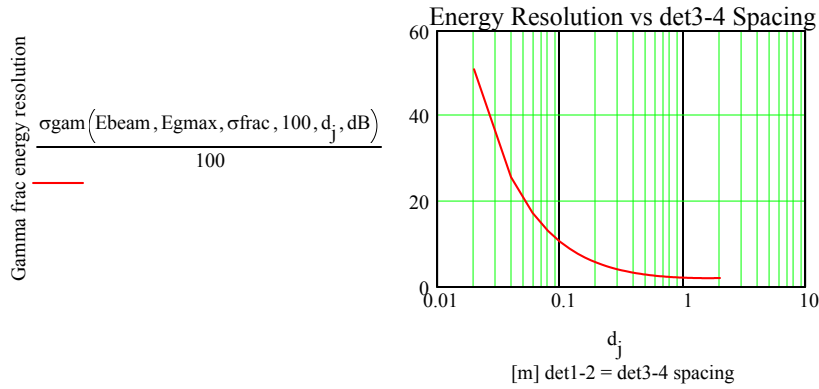
$$\sigma_{\theta si} := \sqrt{2 \cdot \left(\frac{\sigma_{si}}{dA} \right)^2 + 2 \cdot \left(\frac{\sigma_{si}}{dA} \right)^2} \quad \sigma_{\theta si} = 6.986 \cdot 10^{-4}$$

[radians] Bend angle error due to multiple scattering at Ebeam

$$\sigma_{ms} := \frac{13.6}{Ebeam} \cdot \sqrt{X0} \quad \sigma_{ms} = 7.877 \cdot 10^{-5}$$

$$\text{nbins} := 100 \quad i := 0 \dots \text{nbins} - 1 \quad E\gamma_i := E_{\text{gmax}} \cdot \frac{(i+1)}{\text{nbins}} \quad j := 0 \dots 100 \quad d_j := (j+1) \cdot 0.02$$

Find the telescope $\text{det1-det2} = \text{det3-det4} = dA$ spacing that minimizes the gamma energy resolution at 100 Mev by varying dA .



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