Stress Analysis for the Corner Flexure to Grid Attachement Concept

Factors and Allowables:

Factors

$MUF := 1.15$	(Model Uncertainty Factor)
$FS := 1.12$	(Factor of Safety on Qualification loads: Qualification levels are 25% higher than design levels ... giving a factor of 1.4 on design loads, as required)

Allowables

$\sigma_{ult.A286} := 130000psi$	(S-Basis, MIL-HDBK-5H allowable for A286)
$\sigma_{yield.A286} := 85000 \cdot psi$	(S-Basis, MIL-HDBK-5H allowable for A286)
$\sigma_{cy.A286} := 85000psi$	(S-Basis, MIL-HDBK-5H allowable for A286)
$\tau_{ult.A286} := 85000 \cdot psi$	(S-Basis, MIL-HDBK-5H allowable for A286)
$\sigma_{yield.Ti} := 120000psi$	(A-Basis, MIL-HDBK-5H allowable for 6Al-4V Titanium)
$\sigma_{cy.Ti} := 124000psi$	(A-Basis, MIL-HDBK-5H allowable for 6Al-4V Titanium)
$\tau_{ult.Ti} := 79000psi$	(A-Basis, MIL-HDBK-5H allowable for 6Al-4V Titanium)
$\sigma_{bru.1.5} := 214000psi$	(e/D=1.5, B-Basis, MIL-HDBK-5H allowable for 6Al-4V Titanium)
$\sigma_{bru.2} := 276000psi$	(e/D=2.0, B-Basis, MIL-HDBK-5H allowable for 6Al-4V Titanium)
$\sigma_{bry.1.5} := 179000psi$	(e/D=1.5, B-Basis, MIL-HDBK-5H allowable for 6Al-4V Titanium)
$\sigma_{bry.2} := 212000psi$	(e/D=2.0, B-Basis, MIL-HDBK-5H allowable for 6Al-4V Titanium)

Corner Flexure Stress Analysis

Corner Flexure-to-Grid Interface Forces (Taken from Detailed TKR FEM)

$factor := \frac{24}{27}$ (Scale factor for load relief - 3dB Notching of RV spectrum around fundamental frequency)

$P_{corner.lat} := 980 \cdot factor\ N$ $P_{corner.lat} = 195.834\ lbf$

$P_{corner.vert} := 4124 \cdot factor\ N$ $P_{corner.vert} = 824.1\ lbf$

$P_{shear.max} := \sqrt{P_{corner.lat}^2 + P_{corner.vert}^2}$ $P_{shear.max} = 847.048\ lbf$

Torque and Preload Requirements for a #10-32 Screw

$Torque := 30 in \cdot lbf$ (Torque Spec from ...)

$Dia_{10} := 0.19 in$

$K := .2$

Used John Ku's numbers here for consistency between analyses
(Prefer K=0.25 for a typical friction coefficient of .20 between threads per Table 6 of NASA Fastener Design Manual RP-1228)

$Preload_{nom} := \frac{Torque}{K \cdot Dia_{10}}$ (Ref: Equation per NASA Manual RP-1228) $Preload_{nom} = 789.474\ lbf$

Preload Uncertainty for using a Torque wrench is +/-25% per NASA RP-1228 Table 7

$Preload_{min} := Preload_{nom} \cdot .75$ $Preload_{min} = 592.105\ lbf$

$Preload_{max} := Preload_{nom} \cdot 1.25$ $Preload_{max} = 986.842\ lbf$

Preload that will be used in the following calculations (user defined to verify margins against load uncertainty)

$Preload := Preload_{nom}$ $Preload = 789.474\ lbf$

Tensile Stress on Fastener

$A_{stress.1032} := .02 in^2$

$\sigma_{axial.10} := \frac{Preload}{A_{stress.1032}}$ $\sigma_{axial.10} = 3.947 \times 10^4\ psi$

$MS_{yield.10} := \frac{\sigma_{yield.A286}}{FS \cdot MUF \cdot \sigma_{axial.10}} - 1$ $MS_{yield.10} = 0.672$ $\%_{yield} := \frac{\sigma_{axial.10}}{\sigma_{yield.A286}}$

$MS_{ult.10} := \frac{\sigma_{ult.A286}}{FS \cdot MUF \cdot \sigma_{axial.10}} - 1$ $MS_{ult.10} = 1.557$ $\%_{yield} = 46.44\,\%$

Corner Flexure Hoop Stress Calculations (All Four Corners)

Hoop Stress in the Flexure: this analysis looks at the minimum area and assumes that all the axial force is reacted through the smallest area. The preload from the bolt is reacted out through two sides of the cone. The diameter of the inner cone is 0.485" maximum and 0.381" minimum. The angle of the taper is 8 degrees. The lateral reaction force becomes:

$$A_{flex.top} := 0.0475in^2$$

Area 1: The bottom area calculation below includes a 0.5mm chamfer only, versus the top area calculation.

$$A_{flex.bot.1} := 0.0473in^2$$

Area 2: The bottom area calculation below includes a 0.5mm chamfer and 0.5mm reduction to the cross section to account for the dimension changes made by SLAC (Grid & Tracker Lead Engineers).

$$A_{flex.bot.2} := 0.0345in^2$$

$$\theta_{collar} := 8deg$$

$$F_{R.flex} := \frac{\frac{Preload}{2}}{\tan(\theta_{collar})} \qquad F_{R.flex} = 2.809 \times 10^3 lbf$$

The load will be shared equally on both sides of the hole. Therefore, the highest stress will be at the bottom of the corner flexure base. The minimum margin will be:

Margin Calculation using Area 1 above

$$\sigma_{hoop.flex.1} := \frac{\frac{1}{2}F_{R.flex}}{\min(A_{flex.top}, A_{flex.bot.1})} \qquad \sigma_{hoop.flex.1} = 2.969 \times 10^4 psi$$

$$MS_{flex.hoop.1} := \frac{\sigma_{yield.Ti}}{FS \cdot MUF \cdot \sigma_{hoop.flex.1}} - 1 \qquad MS_{flex.hoop.1} = 2.138$$

Margin Calculation using Area 2 Above

$$\sigma_{hoop.flex.2} := \frac{\frac{1}{2}F_{R.flex}}{\min(A_{flex.top}, A_{flex.bot.2})} \qquad \sigma_{hoop.flex.2} = 4.071 \times 10^4 psi$$

$$MS_{flex.hoop.2} := \frac{\sigma_{yield.Ti}}{FS \cdot MUF \cdot \sigma_{hoop.flex.2}} - 1 \qquad MS_{flex.hoop.2} = 1.289$$

Corner Flexure Bearing Stress Analysis (All Four Corners)

The section is tapered, therefore three calculations will be made to estimate the bearing stress and MS at the largest, middle, and smallest diameters, respectively. The load is assumed to be constant along the bearing length.

$D_{taper.max} := .485in$

$D_{taper.min} := .381in$

$D_{taper.mid} := \frac{D_{taper.max} + D_{taper.min}}{2}$ $\qquad D_{taper.mid} = 0.433\,in$

$L_{taper} := .369in$

$e := .2953in$ (e is 0.2953 in w/out dimension change (i.e. original design), and e is 0.2756 in w/ dimension change (i.e. remove 0.5mm from bottom))

$w_L := \frac{P_{corner.vert}}{L_{taper}}$ $\qquad w_L = 2.233 \times 10^3 \frac{lbf}{in}$

Maximum Diameter

Knockdown factors for edge distance <1.5

$eoverD_{max} := \frac{e}{D_{taper.max}}$ $\qquad eoverD_{max} = 0.609$

$\sigma_{bru.all.max} := (eoverD_{max} - .5)\cdot\frac{\sigma_{bru.2}}{(2 - .5)}$ $\qquad \sigma_{bru.all.max} = 2.003 \times 10^4\,psi$

$\sigma_{bry.all.max} := (eoverD_{max} - .5)\cdot\frac{\sigma_{bry.2}}{(2 - .5)}$ $\qquad \sigma_{bry.all.max} = 1.539 \times 10^4\,psi$

Bearing Calculation: Assume a cosine distribution around the hole

$\sigma_{br.act.max} := \frac{4\cdot w_L}{\pi\cdot D_{taper.max}}$ $\qquad \sigma_{br.act.max} = 5.863 \times 10^3\,psi$

$MS_{flex.bru.max} := \frac{\sigma_{bru.all.max}}{FS\cdot MUF\cdot\sigma_{br.act.max}} - 1$ $\qquad MS_{flex.bru.max} = 1.653$

$MS_{flex.bry.max} := \frac{\sigma_{bry.all.max}}{FS\cdot MUF\cdot\sigma_{br.act.max}} - 1$ $\qquad MS_{flex.bry.max} = 1.038$

Median Diameter

Knockdown factors for edge distance <1.5

$$eoverD_{mid} := \frac{e}{D_{taper.mid}} \qquad eoverD_{mid} = 0.682$$

$$\sigma_{bru.all.mid} := \left(eoverD_{mid} - .5\right)\cdot\frac{\sigma_{bru.2}}{(2 - .5)} \qquad \sigma_{bru.all.mid} = 3.349\times 10^{4}\,psi$$

$$\sigma_{bry.all.mid} := \left(eoverD_{mid} - .5\right)\cdot\frac{\sigma_{bry.2}}{(2 - .5)} \qquad \sigma_{bry.all.mid} = 2.572\times 10^{4}\,psi$$

Bearing Calculation: Assume a cosine distribution around the hole

$$\sigma_{br.act.mid} := \frac{4\cdot w_L}{\pi\cdot D_{taper.mid}} \qquad \sigma_{br.act.mid} = 6.567\times 10^{3}\,psi$$

$$MS_{flex.bru.mid} := \frac{\sigma_{bru.all.mid}}{FS\cdot MUF\cdot \sigma_{br.act.mid}} - 1 \qquad MS_{flex.bru.mid} = 2.959$$

$$MS_{flex.bry.mid} := \frac{\sigma_{bry.all.mid}}{FS\cdot MUF\cdot \sigma_{br.act.mid}} - 1 \qquad MS_{flex.bry.mid} = 2.041$$

Minimum Diameter

Knockdown factors for edge distance <1.5

$$eoverD_{min} := \frac{e}{D_{taper.min}} \qquad eoverD_{min} = 0.775$$

$$\sigma_{bru.all.min} := \left(eoverD_{min} - .5\right)\cdot\frac{\sigma_{bru.2}}{(2 - .5)} \qquad \sigma_{bru.all.min} = 5.061\times 10^{4}\,psi$$

$$\sigma_{bry.all.min} := \left(eoverD_{min} - .5\right)\cdot\frac{\sigma_{bry.2}}{(2 - .5)} \qquad \sigma_{bry.all.min} = 3.888\times 10^{4}\,psi$$

Bearing Calculation: Assume a cosine distribution around the hole

$$\sigma_{br.act.min} := \frac{4\cdot w_L}{\pi\cdot D_{taper.min}} \qquad \sigma_{br.act.min} = 7.463\times 10^{3}\,psi$$

$$MS_{flex.bru.min} := \frac{\sigma_{bru.all.min}}{FS\cdot MUF\cdot \sigma_{br.act.min}} - 1 \qquad MS_{flex.bru.min} = 4.265$$

$$MS_{flex.bry.min} := \frac{\sigma_{bry.all.min}}{FS\cdot MUF\cdot \sigma_{br.act.min}} - 1 \qquad MS_{flex.bry.min} = 3.044$$

Check Margins against the new dimension (i.e. 0.5mm reduction)

$e_2 := .2756\text{in}$

(e is 0.2953 in w/out dimension change (i.e. old design), and e is 0.2756 in w/ dimension change (i.e. remove 0.5mm from bottom))

$w_L = 2.233 \times 10^3 \dfrac{\text{lbf}}{\text{in}}$

Maximum Diameter

Knockdown factors for edge distance <1.5

$$eoverD_{max.2} := \frac{e_2}{D_{taper.max}} \qquad eoverD_{max.2} = 0.568$$

$$\sigma_{bru.all.max.2} := \left(eoverD_{max.2} - .5\right)\cdot\frac{\sigma_{bru.2}}{(2 - .5)} \qquad \sigma_{bru.all.max.2} = 1.256 \times 10^4\,\text{psi}$$

$$\sigma_{bry.all.max.2} := \left(eoverD_{max.2} - .5\right)\cdot\frac{\sigma_{bry.2}}{(2 - .5)} \qquad \sigma_{bry.all.max.2} = 9.646 \times 10^3\,\text{psi}$$

Bearing Calculation: Assume a cosine distribution around the hole

$$\sigma_{br.act.max.2} := \frac{4\cdot w_L}{\pi\cdot D_{taper.max}} \qquad \sigma_{br.act.max.2} = 5.863 \times 10^3\,\text{psi}$$

$$MS_{flex.bru.max.2} := \frac{\sigma_{bru.all.max.2}}{FS\cdot MUF\cdot\sigma_{br.act.max.2}} - 1 \qquad MS_{flex.bru.max.2} = 0.663$$

$$MS_{flex.bry.max.2} := \frac{\sigma_{bry.all.max.2}}{FS\cdot MUF\cdot\sigma_{br.act.max.2}} - 1 \qquad MS_{flex.bry.max.2} = 0.277$$

Corner Flexure Tearout Stress Analysis

$$\tau_{tearout.avg} := \frac{P_{corner.vert}}{2\cdot e\cdot L_{taper}} \qquad \tau_{tearout.avg} = 3.781 \times 10^3\,\text{psi}$$

$$MS_{flex.tearout} := \frac{\tau_{ult.Ti}}{FS\cdot MUF\cdot\tau_{tearout.avg}} - 1 \qquad MS_{flex.tearout} = 15.22$$

Conical Collar Stress Analysis (Corners #1, #2 & #3)

$D_{o.collar} := .3428in$

$D_{i.collar} := .199in$

Collar Shear Stress

$$\tau_{collar} := \frac{P_{shear.max}}{\frac{\pi}{4}\cdot\left(D_{o.collar}^{2} - D_{i.collar}^{2}\right)}$$

$\tau_{collar} = 1.384 \times 10^{4}\,psi$

Collar Bending Stress

$L_{bend.collar} := 0.065in$

The conical end is assumed to be guided. It is capable of reacting out a moment through the conical head.

$$M_{collar} := \frac{1}{2}P_{shear.max}\cdot L_{bend.collar}$$

$M_{collar} = 27.529\,in\cdot lbf$

$$c_{collar} := \frac{D_{o.collar}}{2}$$

$c_{collar} = 0.171\,in$

$$I_{collar} := \frac{\pi}{64}\cdot\left(D_{o.collar}^{4} - D_{i.collar}^{4}\right)$$

$I_{collar} = 6.009 \times 10^{-4}\,in^{4}$

$$\sigma_{collar} := \frac{M_{collar}\cdot c_{collar}}{I_{collar}}$$

$\sigma_{collar} = 7.853 \times 10^{3}\,psi$

Margin Calculation using Tension-Shear Interaction Equations

$$MS_{yield.collar} := \frac{1}{\sqrt{\left(\frac{FS\cdot MUF\cdot\sigma_{collar}}{\sigma_{yield.A286}}\right)^{2} + \left(\frac{FS\cdot MUF\cdot\tau_{collar}}{\tau_{ult.A286}}\right)^{2}}} - 1$$

$MS_{yield.collar} = 3.147$

$$MS_{ult.collar} := \frac{1}{\sqrt{\left(\frac{FS\cdot MUF\cdot\sigma_{collar}}{\sigma_{ult.A286}}\right)^{2} + \left(\frac{FS\cdot MUF\cdot\tau_{collar}}{\tau_{ult.A286}}\right)^{2}}} - 1$$

$MS_{ult.collar} = 3.47$

#10-32 Bolt Analysis under Operational Loads (All Four Corners)

Operational loads induce an axial load in the bolt due to the taper in the conical collar. These loads are reviewed here and margins are calculated to determine the operating stress in the bolt. All forces are reacted out as axial loads on the bolt, i.e. there are no shear forces to include in this analysis. Friction is neglected. Corner #4 will see these axial loads and this analysis is acceptable for the threaded portion of the pin only.

Note: The maximum load and axial stress area is the same for all four corners. Therefore, the margins are also the same.

$$P_{screw.axial} := P_{shear.max} \cdot \tan\left(\theta_{collar}\right) \qquad P_{screw.axial} = 119.045\,\text{lbf}$$

$$P_{tot.screw} := P_{screw.axial} + Preload_{nom} \qquad P_{tot.screw} = 908.519\,\text{lbf}$$

$$\sigma_{tot.screw} := \frac{P_{tot.screw}}{A_{stress.1032}} \qquad \sigma_{tot.screw} = 4.543 \times 10^{4}\,\text{psi}$$

$$MS_{yield.screw} := \frac{\sigma_{yield.A286}}{FS \cdot MUF \cdot \sigma_{tot.screw}} - 1 \qquad MS_{yield.screw} = 0.453$$

$$MS_{ult.screw} := \frac{\sigma_{ult.A286}}{FS \cdot MUF \cdot \sigma_{tot.screw}} - 1 \qquad MS_{ult.screw} = 1.222$$

1/4" Pin Stress Analysis (Corner #4)

$$D_{pin} := .249\text{in}$$

$$A_{pin} := \frac{\pi}{4} \cdot D_{pin}^{2} \qquad A_{pin} = 0.049\,\text{in}^{2}$$

Pin Shear Stress

$$\tau_{pin} := \frac{P_{shear.max}}{A_{pin}} \qquad \tau_{pin} = 1.739 \times 10^{4}\,\text{psi}$$

Pin Bending Stress

$$L_{bend.pin} := 0.065\text{in}$$

$$M_{pin} := \frac{1}{2} P_{shear.max} \cdot L_{bend.pin} \qquad M_{pin} = 27.529\,\text{in} \cdot \text{lbf}$$

$$c_{pin} := \frac{D_{pin}}{2} \qquad c_{pin} = 0.125\,\text{in}$$

$$I_{pin} := \frac{\pi}{64} \cdot \left(D_{pin}^{4}\right) \qquad I_{pin} = 1.887 \times 10^{-4}\,\text{in}^{4}$$

$$\sigma_{bend} := \frac{M_{pin} \cdot c_{pin}}{I_{pin}} \qquad \sigma_{bend} = 1.816 \times 10^{4}\,\text{psi}$$

Pin Axial Stress (from operational loads)

$P_{axial} := P_{tot.screw}$ $\qquad$ $P_{axial} = 908.519\,lbf$

$\sigma_{axial} := \frac{P_{axial}}{A_{pin}}$ $\qquad$ $\sigma_{axial} = 1.866 \times 10^4\,psi$

Tension on Pin

$\sigma_{tension} := \sigma_{bend} + \sigma_{axial}$ $\qquad$ $\sigma_{tension} = 3.682 \times 10^4\,psi$

Margin Calculation using Tension-Shear Interaction Equations

$$MS_{yield.pin} := \frac{1}{\sqrt{\left(\frac{FS \cdot MUF \cdot \sigma_{tension}}{\sigma_{yield.A286}}\right)^2 + \left(\frac{FS \cdot MUF \cdot \tau_{pin}}{\tau_{ult.A286}}\right)^2}} - 1$$

$MS_{yield.pin} = 0.621$

$$MS_{ult.pin} := \frac{1}{\sqrt{\left(\frac{FS \cdot MUF \cdot \sigma_{tension}}{\sigma_{ult.A286}}\right)^2 + \left(\frac{FS \cdot MUF \cdot \tau_{pin}}{\tau_{ult.A286}}\right)^2}} - 1$$

$MS_{ult.pin} = 1.222$

Hoop Stresses in Inner Turn Plug (Corner #4)

The Inner Turn Plug will react the radial forces through hoop stresses in the walls. The Inner Turn Plug will compress. The Outer Turn Plug will see reaction forces from the Inner Turn Plug and the Flexure. The Outer Turn Plug will not see significant hoop stresses and react only compression forces through the thickness. This analysis is very preliminary and is being evaluated in greater detail.

$A_{min.plug} := 1.12 \cdot 0.0143 in^2$ $\qquad$ $A_{min.plug} = 0.016\,in^2$

$F_{R.flex} = 2.809 \times 10^3\,lbf$

$\sigma_{hoop.plug} := \frac{\frac{1}{2} F_{R.flex}}{\left(A_{min.plug}\right)}$ $\qquad$ $\sigma_{hoop.plug} = 8.768 \times 10^4\,psi$

$MS_{plug.hoop} := \frac{\sigma_{cy.Ti}}{FS \cdot MUF \cdot \sigma_{hoop.plug}} - 1$ $\qquad$ $MS_{plug.hoop} = 0.098$

Eccentric Washer Design

Pros: Easy Cleanup/Preparation for Integration
Stiff Connection

Cons: Risk of Inner Plug getting wedged into place ... May Cause Serious Heartache
More Parts
Induced Stress - Humans Do Alignment
Induced Stresses on Flexures
Cyclic axial load on bolts - operations
Amplitude Dependent Stiffness Effects from 0.065" Offset between Grid and Taper
Higher Cost to Fabricate
Custom Hardware Components

Epoxy in Remaining Screws

Pros: No Induced Stress from Human Alignment/Friction of Joint
Representative Joint to FEA
Simple Design
Simple to Implement
Easy to Retrofit to EM Tower
Greater Bearing Surface
Give Back 0.5mm on Bottom
Lower Cost to Fabricate
Off-the-Shelf Hardware

Cons: Messy Cleanup
Epoxy more compliant than metal (overall stiffness drop, not amplitude dependent)

Alternate Solution: Use the side flexures to locate the TKR Tower. Install a 1/4" pin into a side flexure pin location (edge #1) to constrain Z and Y. Install a 1/4" pin in the opposing side flexure pin location (edge #3) to constrain Z and Y. Install a final 1/4" pin into one of the adjacent side flexure pin locations (edge #2) to constrain Z and X. This provides six constraints for six degrees of freedom (3-2-1 constraint). Avoid slop by making the pins 0.0002" to 0.0004" oversized, causing a slight interference fit. Do the same to the grid. Epoxy in Remaining Nine Screws.

Pros: Simple Design
Easy to Retrofit
No Induced Stress on Bottom Tray
Representative Joint to FEA
Greater Bearing Surface
Give Back 0.5mm on Bottom
Lowest Cost to Fabricate
Off-the-Shelf Hardware Throughout

Cons: Messy Cleanup
Epoxy more compliant than metal (overall stiffness drop, not amplitude dependent)