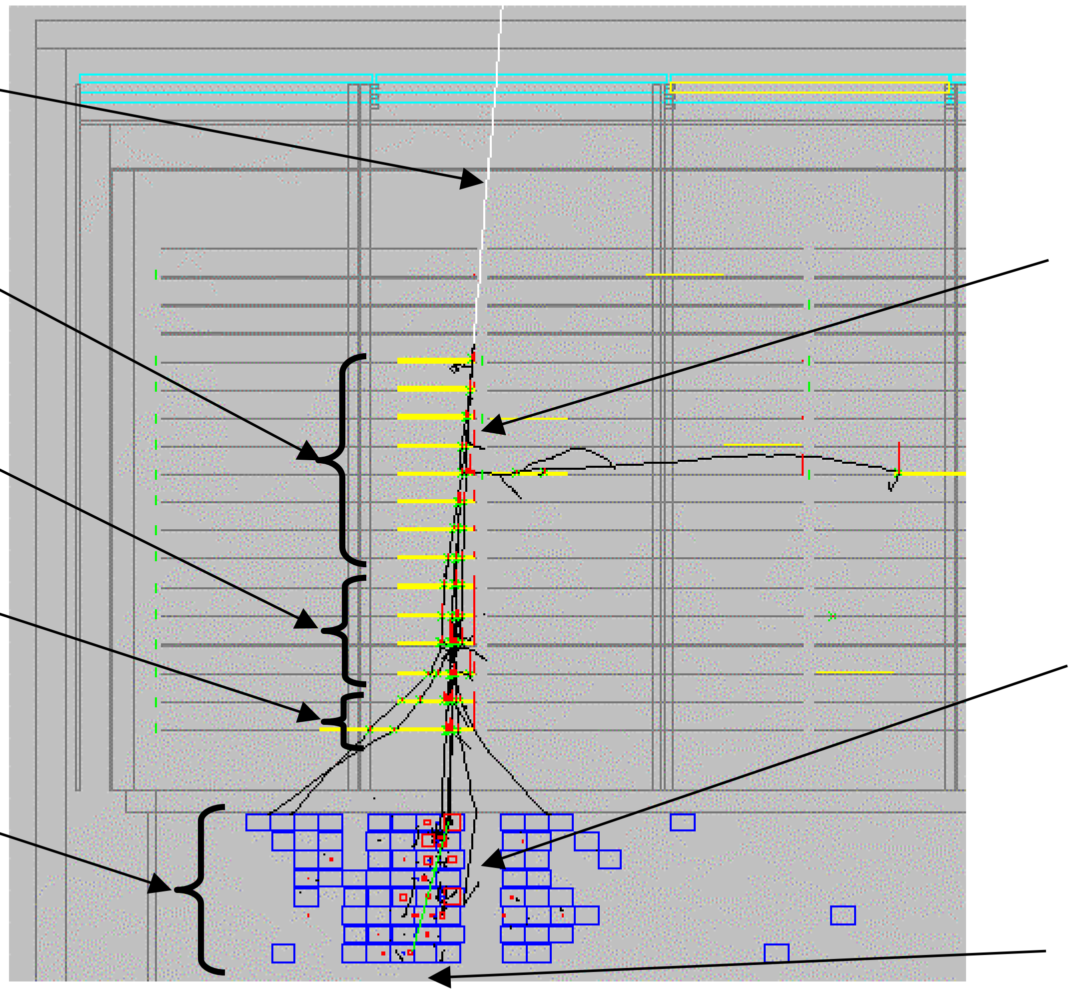
GLAST Energy

or Humpty-Dumpty's Revenge

- A Statement of the Problem
- Divide and Conquer strategy
- Easiest: Leakage (Depth) Correction
- Next Hardest: Tracker Sampling (Details Matter)
- Hardest: Edges (Tracker & Calorimeter)
- Present status

GLAST's Fracture Energy



Divide and Conquer

BIG ASSUMPTION:

THE PROBLEM IS FACTORIZABLE

5 TERMS in ALL!

$$E_{TOT} = E_{TKR} + E_{CAL} \quad \begin{cases} E_{TRK} = E_{TKR\text{-}HITS} * F_{TKR\text{-}EDGE} \\ E_{CAL} = E_{CAL\text{-}XTALS} * F_{CAL\text{-}EDGE} * F_{CAL\text{-}LEAK} \end{cases}$$

$E_{TKR\text{-}HITS}$ is derived from counting tracker hits

$F_{TKR\text{-}EDGE}$ is calculated depending on proximity of Hits to Tower Edges

$E_{CAL\text{-}XTALS}$ is derived from CAL Diode Output

$F_{CAL\text{-}EDGE}$ is calculated depending on proximity of Hits to Tower Edges

$F_{CAL\text{-}LEAK}$ is calculated from the Shower Shape

No. 1: CAL Energy Leakage

Shower Shape model from Wallet Card:

$$\frac{dE}{d(bt)} = E_0 \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)}$$

The numerator is just the integrand of the Γ function which on the interval $[0, \infty]$ = (a-1)! = Γ(a)

b is a scale parameter that is ~ constant with Energy and *b ~ .5*

From this the expectation value of *t* (Energy Centroid) is

$$\langle t \rangle = \frac{\int_0^\infty t (bt)^{a-1} e^{-bt} dt}{\int_0^\infty (bt)^{a-1} e^{-bt} dt} = \frac{1}{b} \frac{\Gamma(a+1)}{\Gamma(a)} = \frac{a}{b}$$

BUT.... We don't have an infinitely deep Calormeter!

ENERGY Leakage (Part II)

Finite Calorimeter: $\Gamma(a) \Rightarrow \Gamma(a, t_{max})$ (This is the Incomplete Γ Function)

As such, we can't write down in closed form the relationship between a and $<t>$

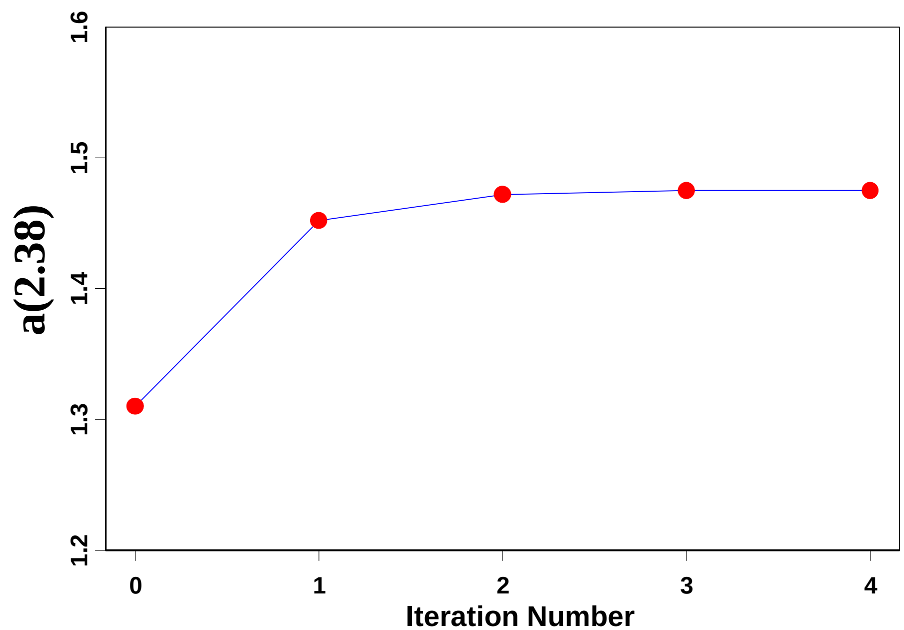
But... we can iteratively solve for *a* given the observed $<t>$

Specifically:

$$a_{i+1} = a_0 \frac{1 - \frac{\Gamma(a_i, bt_{MAX})}{\Gamma(a_i)}}{1 - \frac{\Gamma(a_i + 1, bt_{MAX})}{\Gamma(a_i + 1)}}$$

where $a_0 = <t_{OBS}>*b$

And fortunately it converges quickly!



THERE IS NO FITTING!
THE ENERGY CENTRIOD GIVES THE CORRECTION!

ENERGY Leakage (Part III)

Finally: $$F_{CAL-LEAK} = \frac{\Gamma(a)}{\Gamma(a, bt_{MAX})}$$

The DEVIL is in the DETAILS!

1) The previously thought 20% gain error in the CAL was wrong. When Leakage is included, energies past ~ 500 MeV are over estimated by numbers approaching 20%. It seems there is a ~ 20 MeV pedistal instead.....

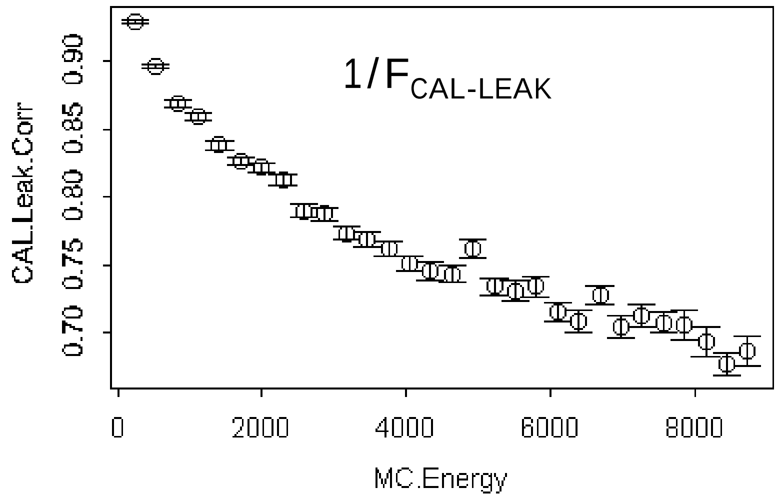
2) You must include the Tracker contributions (t_{TKR}) when computing $\langle t_{OBS} \rangle$

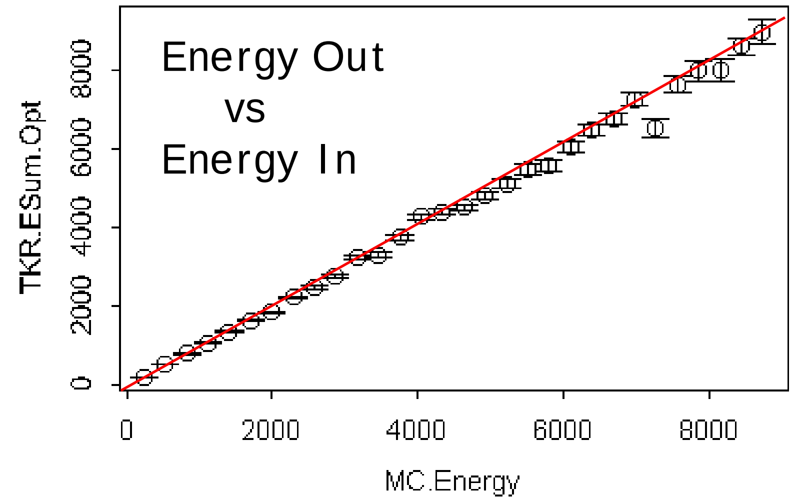
3) In computing the t_{MAX} from arclengths, be aware that there is ~ 9mm (front-to-back)of Carbon material giving the calorimeter an effective radiation length of 19.7 mm (CsI is 18.5mm).

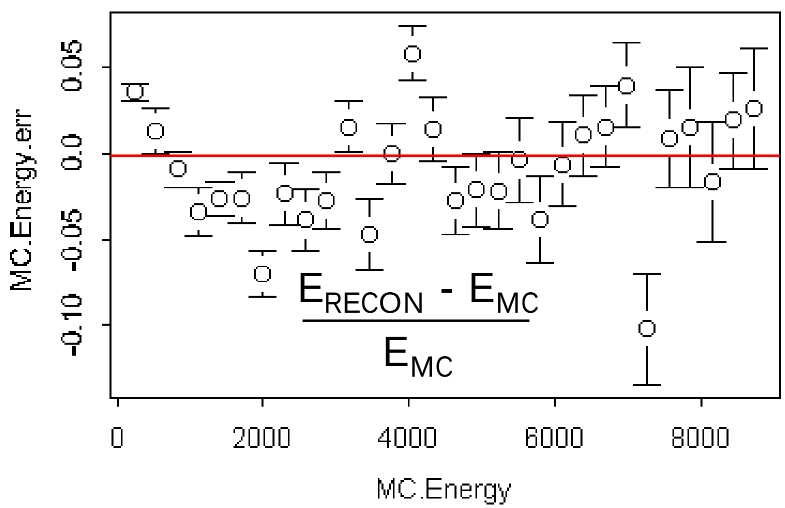
4) Obviously t_{MAX} must include t_{TKR} as well

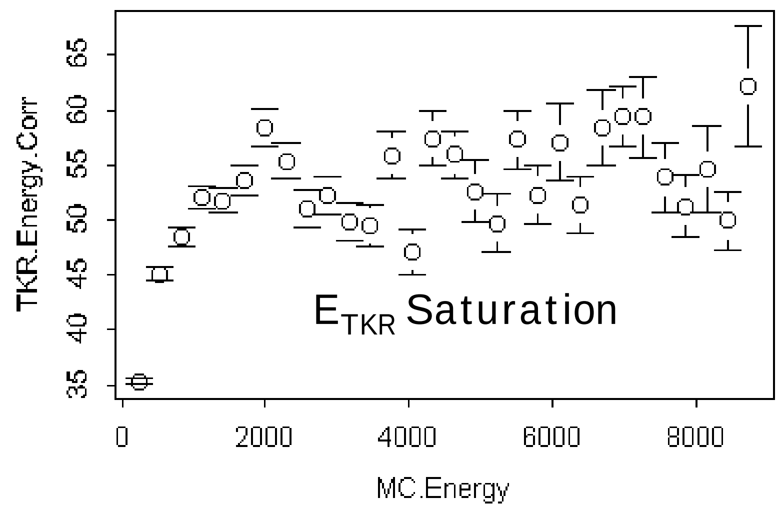
5) The existing formula for the b parameter (see CalRecon) gives much to small values. Presently using b = .55

Energy Leakage Results





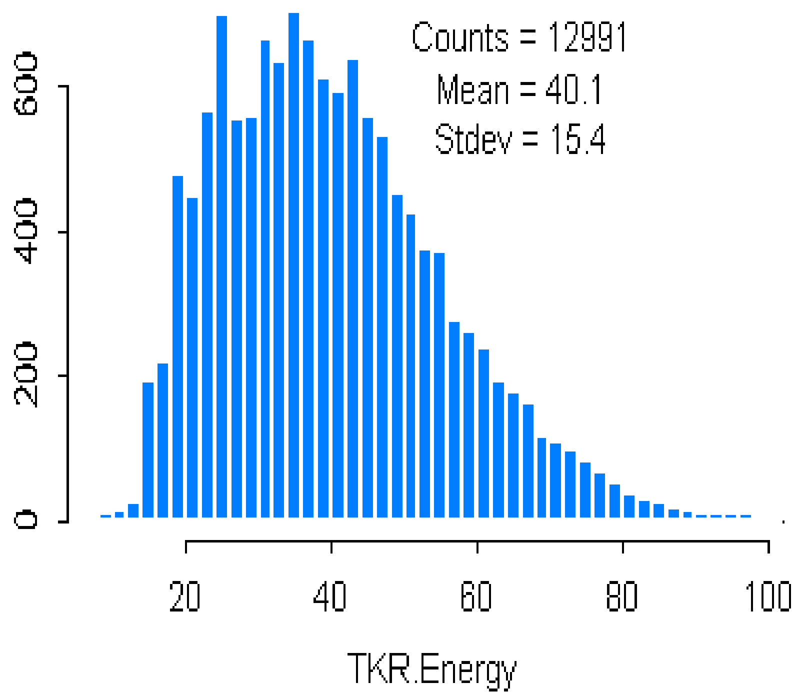


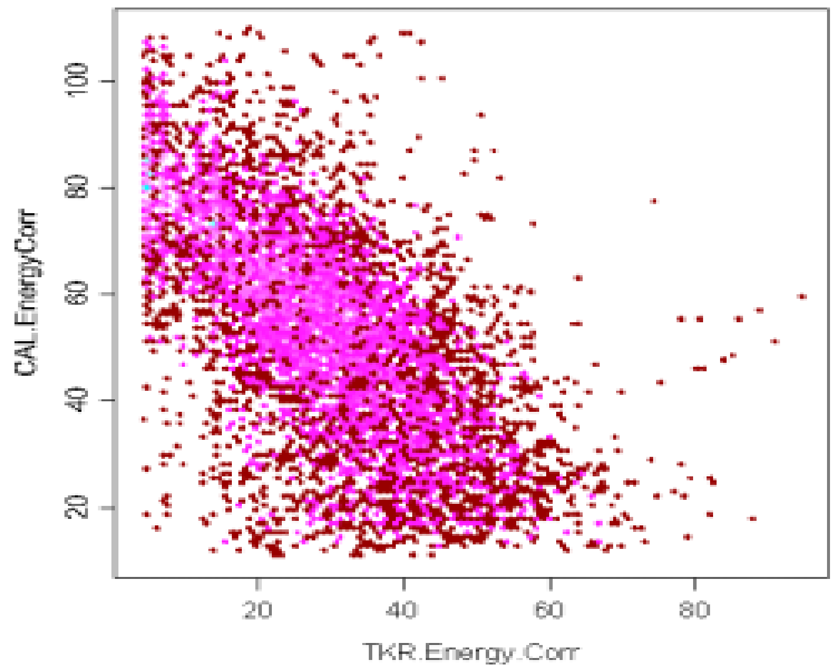


No. 2: Tracker Energy

Tracker Energy becomes increasing important at low energies –

Example – 100 MeV γ's within 5° of Inst. Axis.





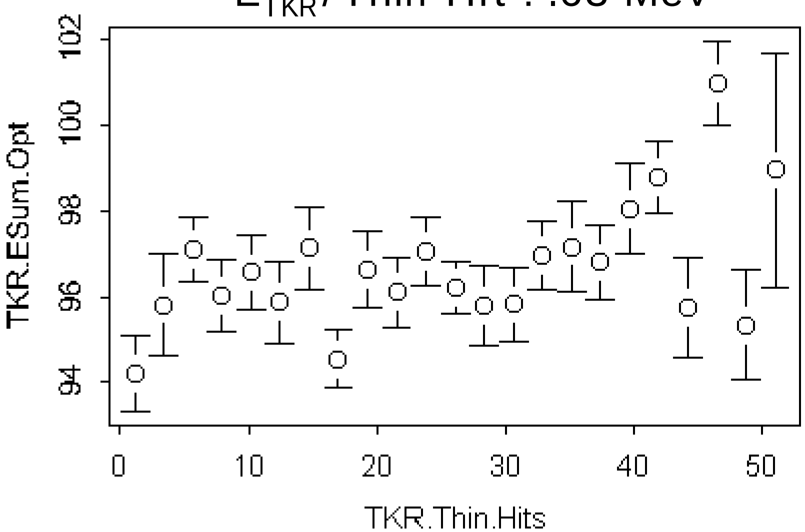
There are 3 pieces to balance:
- Thin Radiator Hits
- Thick Radiator Hits
- Blank Radiator Hits

Tracker Energy – Component Balancing

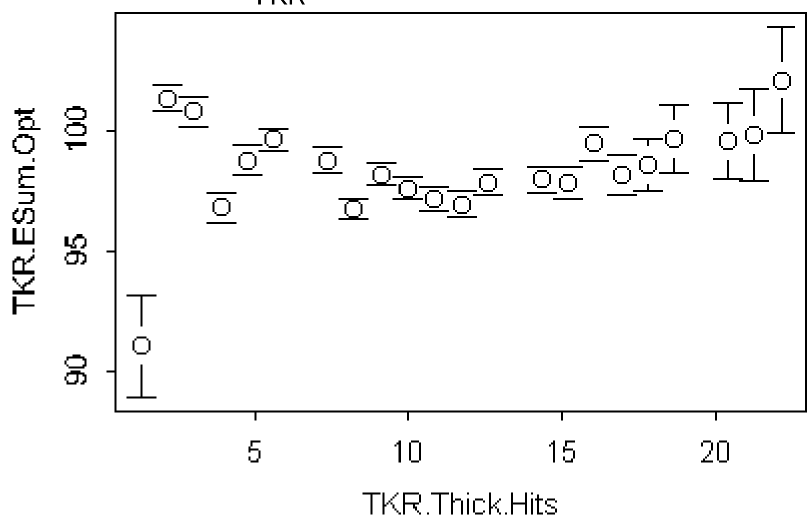
Issue: How much energy to ascribed per hit?
Depends on Layer due to various radiator thicknesses

Goal: To make energy determination independent of the number of hits in any specific layer

E_{TKR}/Thin-Hit : .63 MeV



E_{TKR}/Thick-Hit: 1.67 MeV



Ratio: Thick/Thin = 2.64 Expected 4.3 from radiators

WHY?

In addition: Blank-Hits need .65 MeV/Hit – Probably due to material between TKR & CAL

No. 3: Edge Corrections – The Hardest Part

Basics: Transverse Shower Model
Circular with a Radially dependent distribution
Radius given ~ Moliere Radius modulo log(E) dep.

Longitudinal Shower Model
Cone – saturating to a Cylinder at Shower Max.

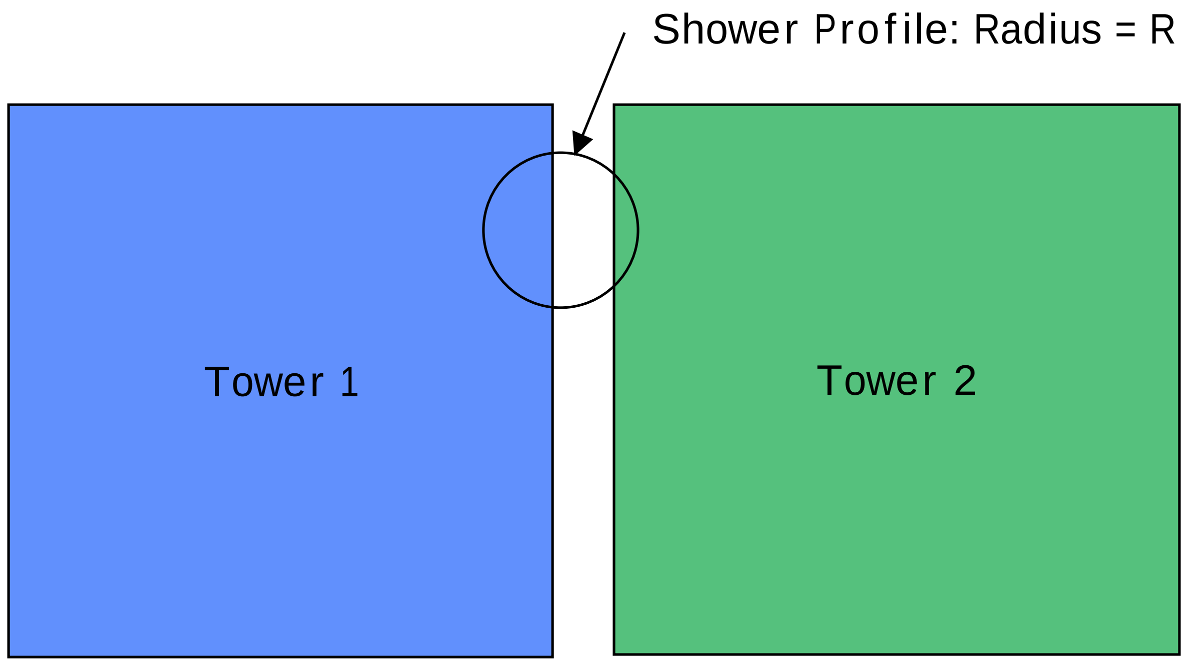
Effect of Edges and Gaps: Loss of Observed Energy

Model Correction: Estimate the lost Active Volume
GLAST is Layers – do estimate layer-by-layer
Treat Layers as thin-sheets
Sum of Layers Approximates 3D Integral

Magnitude of the Effect:
100 MeV ~ 1.5
10000MeV ~ 20.

So... ITS BIG and Energy Dependent!

Edge Corrections – Formalism



Area of a Cord-Defined Slice:

$$A(r,y) = \pi r^2 \left(\frac{1}{2} - \frac{y}{\pi r}\sqrt{1-\frac{y^2}{r^2}} - \frac{\sin^{-1}(\frac{y}{r})}{\pi}\right)$$

where y is the distance to the edge (y goes from 0 to r)

Edge Corrections – Application

So the Area of the Active Areas in general will have 2 Pieces:

$$F_{EDGE} = (A_{TOWER\_1}(r, y_1) + A_{TOWER\_2}(r, y_2))/\pi r^2$$

Given a shower axis, y can be computed at each plane (in z)
As such a psuedo-3D correction can be computed.

To account for radial dependence – divide into 2 Bins
Core – r similar to Moliere radius
Fringe – r ~ 2 x Moliere radius

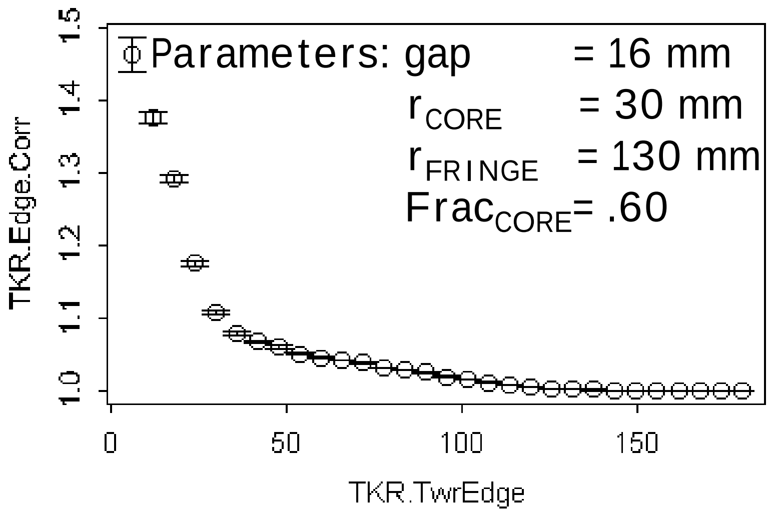
Apply this to the Tracker and to the Calorimeter
(They will have different r's and strategies!)

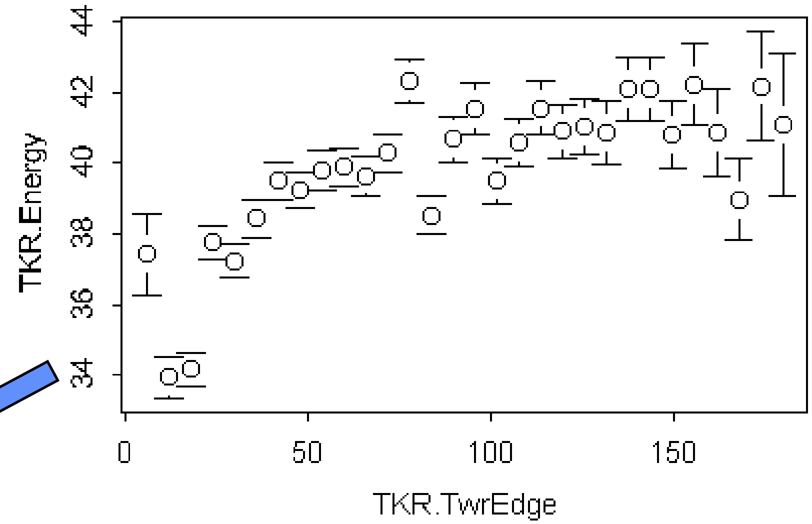
TRACKER:

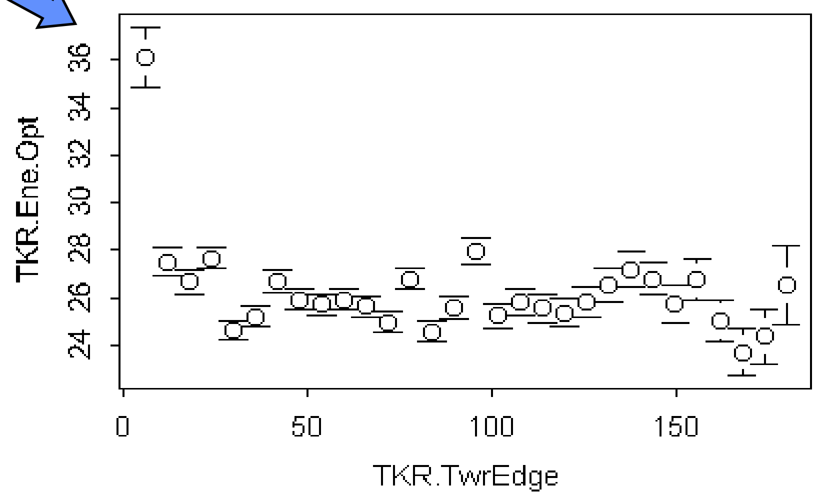
1) Effective radiation length is very large:
Thin section ~ 71 cm / Thick section ~ 16 cm
2) Start of shower – r is much smaller the $r_{Moliere}$
use "fitted value"
3) Plane-to-plane fluctuations in hits make individual plane corrections un-workable. Integrate correction weighted by plane rad. lens. an apply globally.

Tracker Edge Corrections

100 MeV γ's within 5^{o} of Inst. Axis



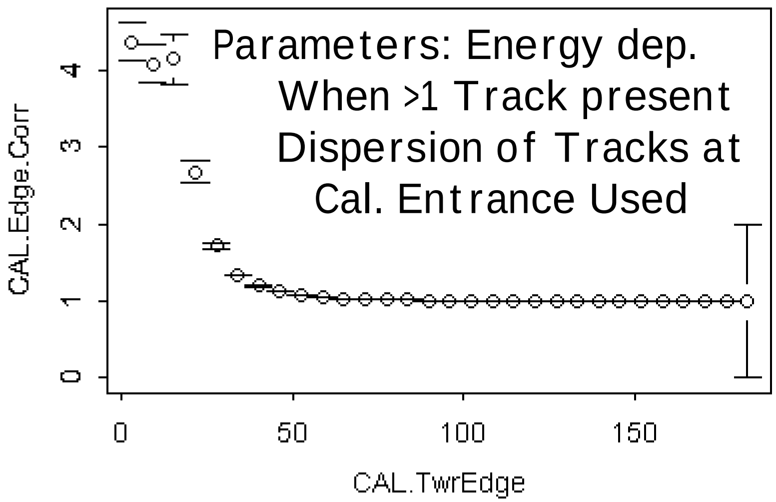


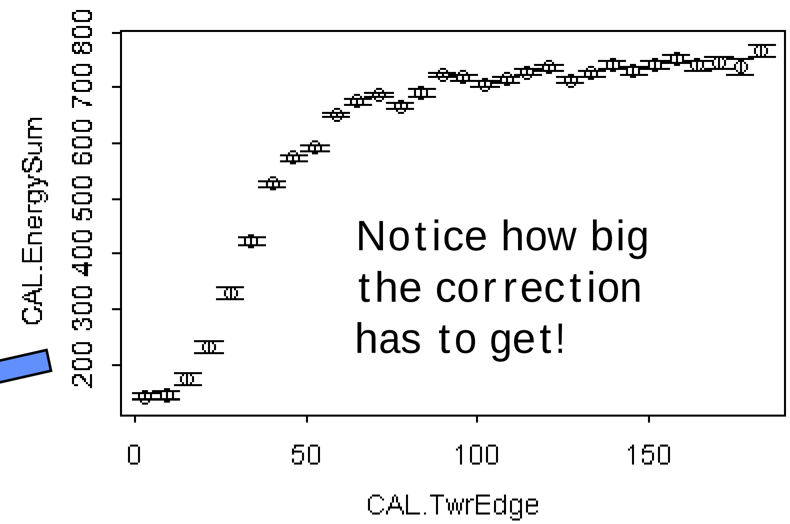


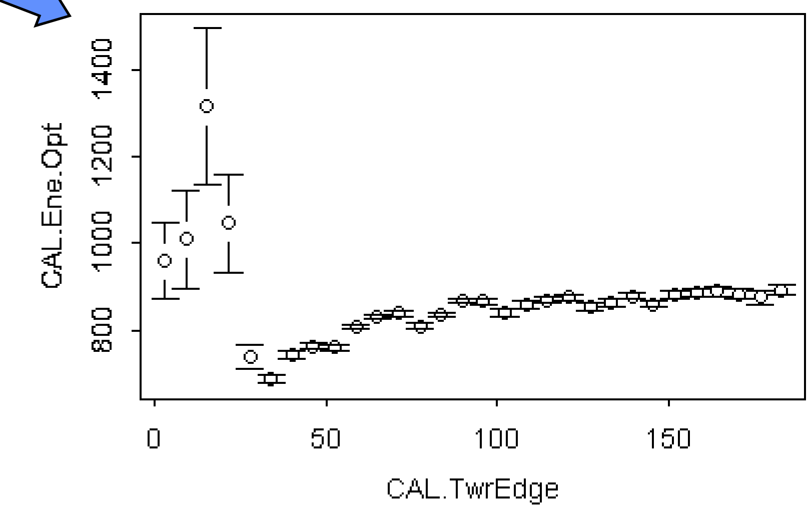
Note: "Over-shoot" near y = 0
Its real – more radiation
lengths here! (Present
correction is too small!)

Calorimeter Edge Corrections

1000 MeV γ's within 5° of Inst. Axis





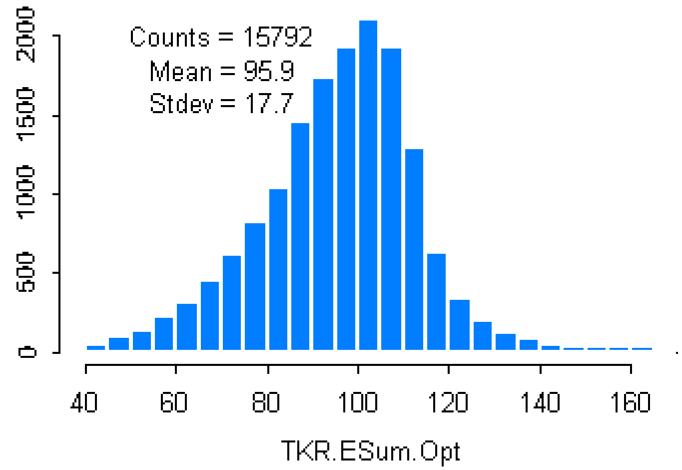


Correction is Problematic:
- Observe ~ 150 MeV near the Edge
- Wind up multiplying it by ~ 5
- BUT... there are alot of Events Here!
  (Within 40 mm of the Edge)

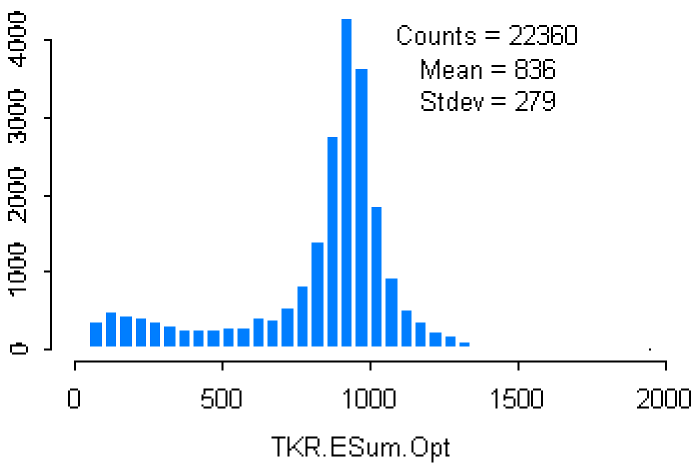
Results to Date

Conclusions:

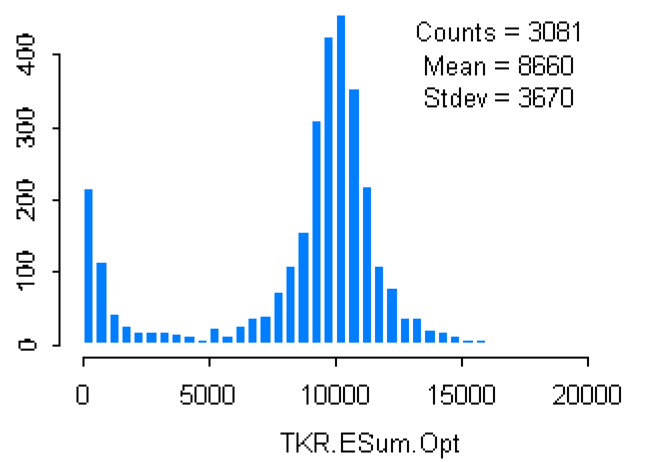
1) It is possible to put GLAST Energy back together
2) Tuning the parameters is a slow process due to all the interlocking pieces
3) Off axis – still to be explored!



FWHM = 35 MeV (σ = 14.8%)



FWHM = 190 MeV (σ = 8.0%)



FWHM = 2200 MeV (σ = 9.3%)