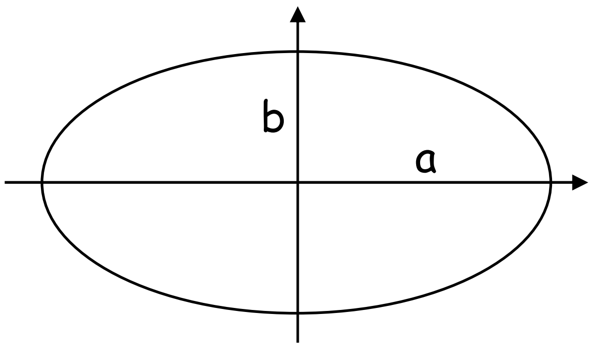
Covariance & GLAST

Agenda

- Review of Covariance
- Application to GLAST
- Kalman Covariance
- Present Status

Review of Covariance

Ellipse



Take a circle – scale the x & y axis:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Rotate by θ :

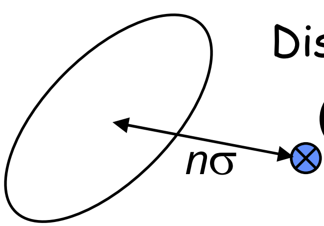
$$x \rightarrow x\cos(\theta) + y\sin(\theta)$$
$$y \rightarrow y\cos(\theta) - x\sin(\theta)$$

Results:

$$x^2\left(\frac{\cos^2(\theta)}{a^2} + \frac{\sin^2(\theta)}{b^2}\right) + 2xy\cos(\theta)\sin(\theta)\left(\frac{1}{a^2} - \frac{1}{b^2}\right) + y^2\left(\frac{\sin^2(\theta)}{a^2} + \frac{\cos^2(\theta)}{b^2}\right) = 1$$

Rotations mix x & y. Major & minor axis plus rotation angle θ complete description.

Error Ellipse described by Covariance Matrix:



Distance between a point with an error and another point measured in σ's:

$(n\sigma)^2 = r^T C^{-1} r$ where $r = (\vec{x} - \bar{x})$ and

$$C^{-1} = Inverse(C) = \begin{bmatrix} C^{-1}_{xx} & C^{-1}_{xy} \\ C^{-1}_{yx} & C^{-1}_{yy} \end{bmatrix}$$ and $C^{-1}_{xy} = C^{-1}_{yx}$

} Simply weighting the distance by $1/\sigma^2$

Review 2

Multiplying it out gives:

$$(n\sigma)^2 = r^T C^{-1} r = (x, y)\begin{bmatrix} C^{-1}_{xx} & C^{-1}_{xy} \\ C^{-1}_{xy} & C^{-1}_{yy} \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = x^2 C^{-1}_{xx} + 2xyC^{-1}_{xy} + y^2 C^{-1}_{yy}$$

Where I take $\bar{x} = 0$ without loss of generality.

This is the equation of an ellipse! Specifically for 1 σ error ellipse ($n\sigma = 1$) we identify:

$$C^{-1}_{xx} = \frac{\cos^2(\theta)}{a^2} + \frac{\sin^2(\theta)}{b^2} \qquad C^{-1}_{yy} = \frac{\sin^2(\theta)}{a^2} + \frac{\cos^2(\theta)}{b^2} \qquad C^{-1}_{xy} = \sin(\theta)\cos(\theta)(\frac{1}{a^2} - \frac{1}{b^2})$$

and $C^{-1} = \frac{1}{\det(C)}\begin{bmatrix} C_{yy} & -C_{xy} \\ -C_{xy} & C_{xx} \end{bmatrix}$ where $\det(C) = (C_{xx}C_{yy} - C^2_{xy})$

And the correlation coefficient is defined as: $r^2 = \frac{C^2_{xy}}{C_{xx}C_{yy}}$

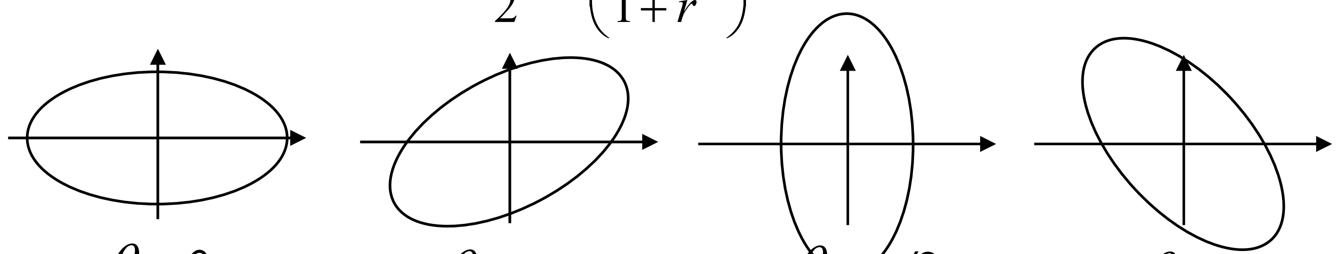
Summary: The inverse of the Covariance Matrix describes an ellipse where the major and minor axis and the rotation angle map directly onto its components!

Review 3

Let the fun begin! To disentangle the two descriptions consider

$$A = \frac{C_{xy}^{-1}}{C_{xx}^{-1} + C_{yy}^{-1}} = \frac{-C_{xy}}{C_{xx} + C_{yy}} = \frac{\cos(\theta)\sin(\theta)(b^2 - a^2)}{a^2 + b^2}$$

thus $A = \frac{\sin(2\theta)}{2}\left(\frac{1-r^2}{1+r^2}\right)$ where $r = a/b$



$\theta = 0$ $\theta = \pi/4$ $\theta = \pi/2$ $\theta = 3\pi/4$

Also det(C) yields (with a little algebra & trig.):

$$a \cdot b = \sqrt{\det(C)}$$

Now we're ready to look at results from GLAST!

Covariance Matrix from Kalman Filter

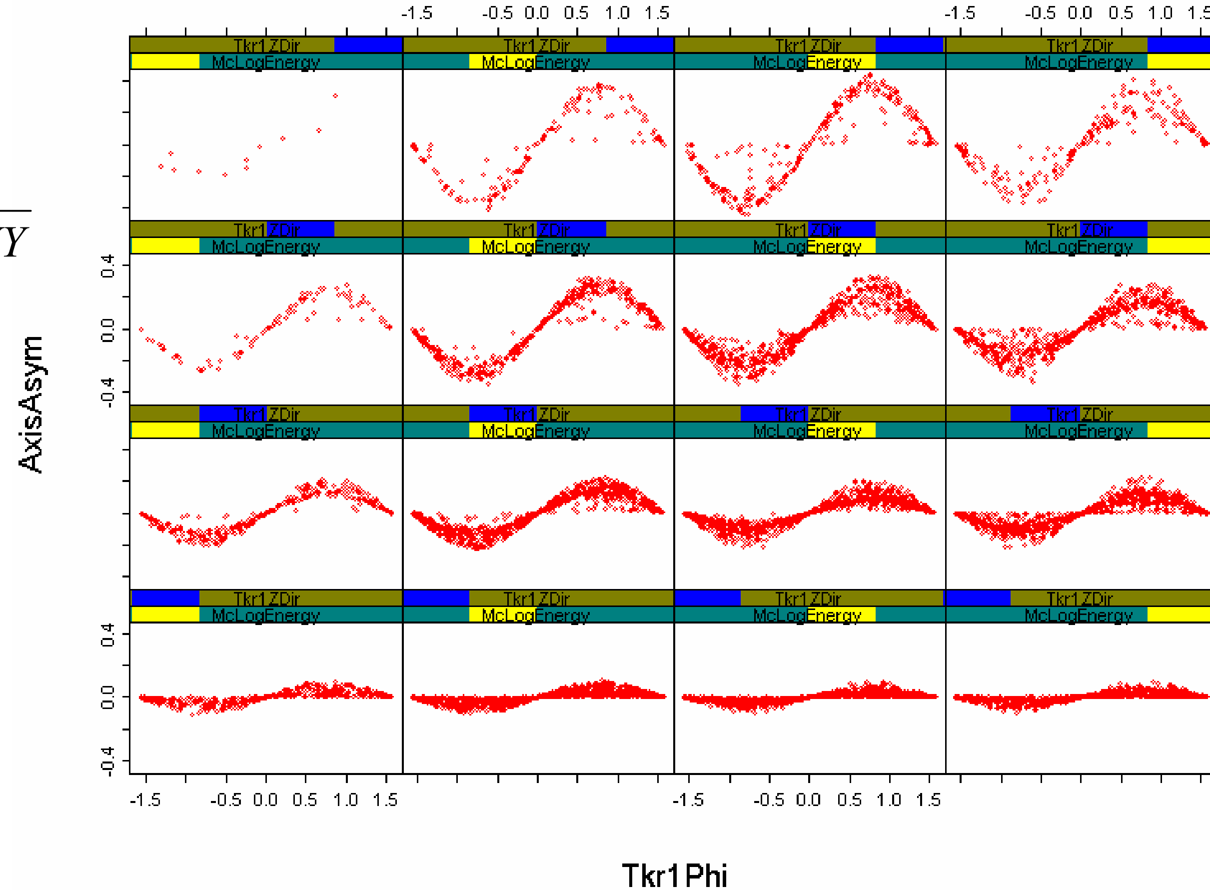
Results shown for

$$AxisAsym = -\frac{Tkr1SXY}{Tkr1SXX + Tkr1SYY}$$

Binned in
$\cos(\theta)$ and $\log_{10}(E_{MC})$

Recall however that KF gives us C in terms of the track slopes *Sx* and *Sy*.

AxisAsym grows like
$1/\cos^2(\theta)$



Peak amplitude ~ .4

$$A(\max) = \frac{1}{2}\frac{(1-r^2)}{(1+r^2)} \Rightarrow r^2 = \frac{1-2A(\max)}{1+2A(\max)} \Rightarrow r = \frac{b}{a} = \sqrt{\frac{1-2\cdot.4}{1+2\cdot.4}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

Relationship between Slopes and Angles

For functions of the estimated variables the usual prescriptions is:

$\sigma_f^2 = \sum_{variables} \sigma_i^2 (\frac{\partial f}{\partial x_i})^2$ when the errors are uncorrelated. For correlated errors this becomes $\sigma_f^2 = \sum_{i,j} \sigma_i (\frac{\partial f}{\partial x_i})(\frac{\partial f}{\partial x_j}) \sigma_j = (\frac{\partial f}{\partial x_i}) C_{i,j} (\frac{\partial f}{\partial x_j})$

and reduces to the uncorrelated case when $C_{i,j} = \sigma_i^2 \delta_{i,j}$

The functions of interest here are:

$$\cos(\theta) = \frac{1}{\sqrt{1 + S_x^2 + S_y^2}} \quad \text{and} \quad \tan(\phi) = -\frac{S_y}{S_x}$$

A bit of math then shows that:

$$\sigma_\theta^2 = \cos^4(\theta)\left(\cos^2(\phi) C_{xx} + 2\sin(\phi)\cos(\phi) C_{xy} + \sin^2(\phi) C_{yy}\right)$$

and

$$\sigma_\phi^2 = \frac{1}{\tan^2(\theta)}\left(\sin^2(\phi) C_{xx} + 2\sin(\phi)\cos(\phi) C_{xy} + \cos^2(\phi) C_{yy}\right)$$

Angle Errors from GLAST

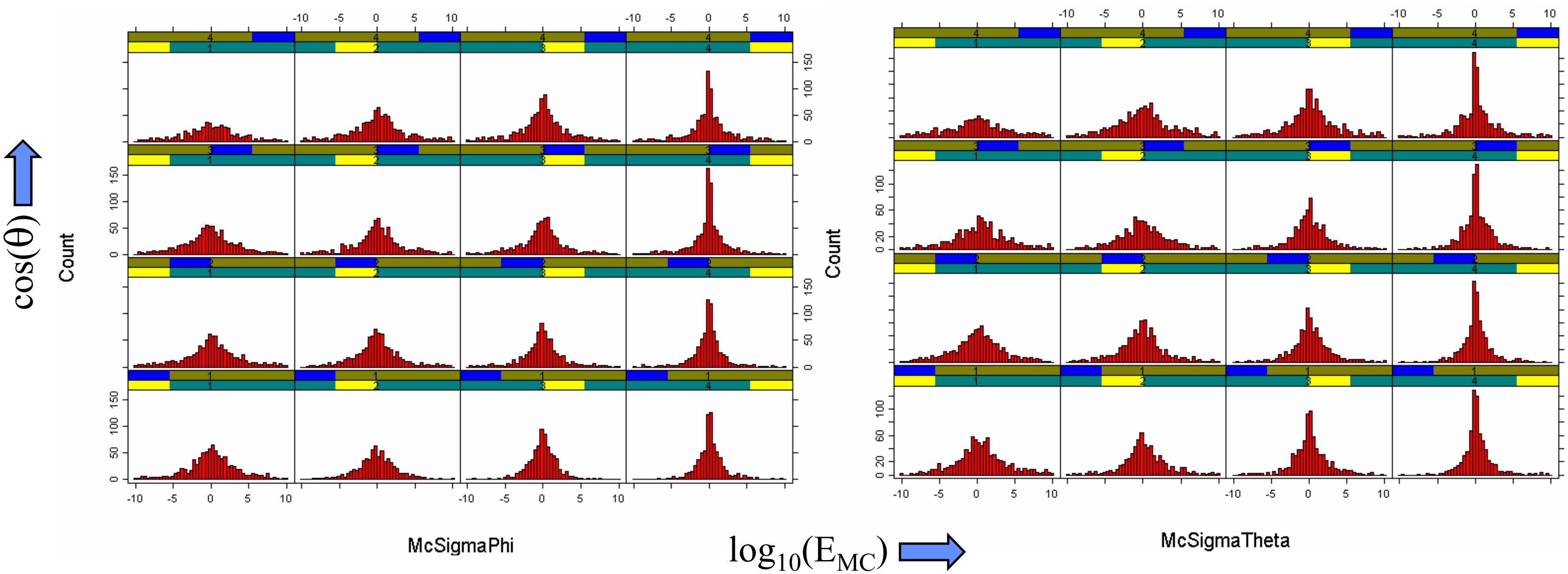
σ_ϕ has a divergence at $\theta = 0$. However $\sigma_\phi \sin(\theta)$ cures this.

σ_θ decreases as $\cos^2(\theta)$ - while $\sigma_\phi \sin(\theta)$ increases as $\frac{1}{\cos(\theta)}$

We also expect the components of the covariance matrix to increase as

$\frac{1}{\cos(\theta)}$ due to the dominance of multiple scattering.

Plot measured residuals in terms of Fit σ's (e.g. $\frac{\theta_{meas} - \theta_{MC}}{\sigma_{FIT}}$)



Angle Errors 2

What's RIGHT:

1) $\cos(\theta)$ dependence
2) Energy dependence in Multiple Scattering dominated range

What's WRONG:

1) Overall normalization of estimated errors (σ_{FIT})
   - off by a factor of ~ 2.3!!!
2) Energy dependence as measurement errors begin to dominate
   - discrepancy goes away(?)

Both of these correlate with with the fact that the fitted χ^2's are much larger then 1 at low energy (expected?).

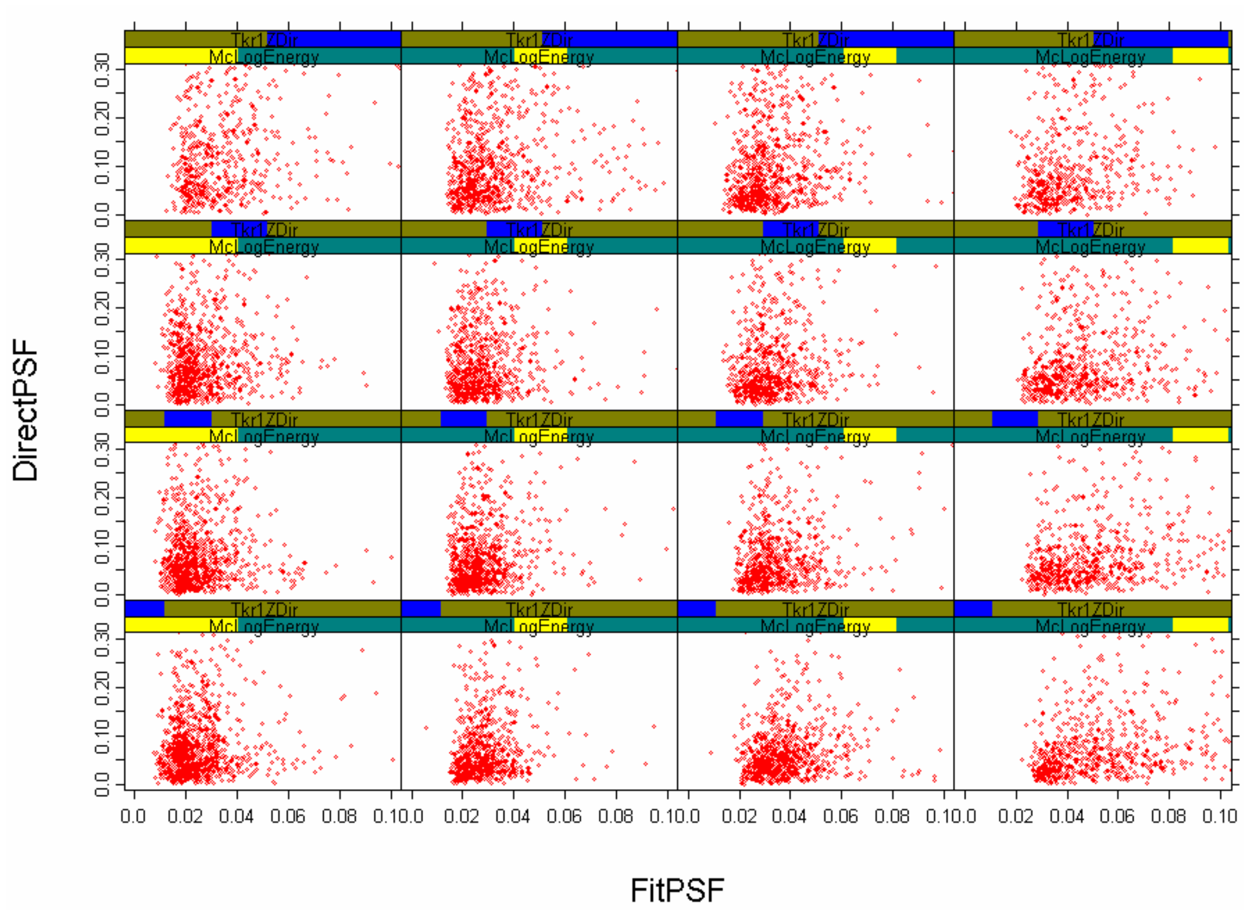
How well does the Kalman Fit PSF model the event to event PSF?

Angle Errors 3

Comparison of

Event-by-Event PSF
vs
FIT Parameter PSF

(Both Energy Compensated)



Difficult to assess
level of correlation

- probably not zero

- approximately same
  factor of 2.3

Angle Errors: Conclusions

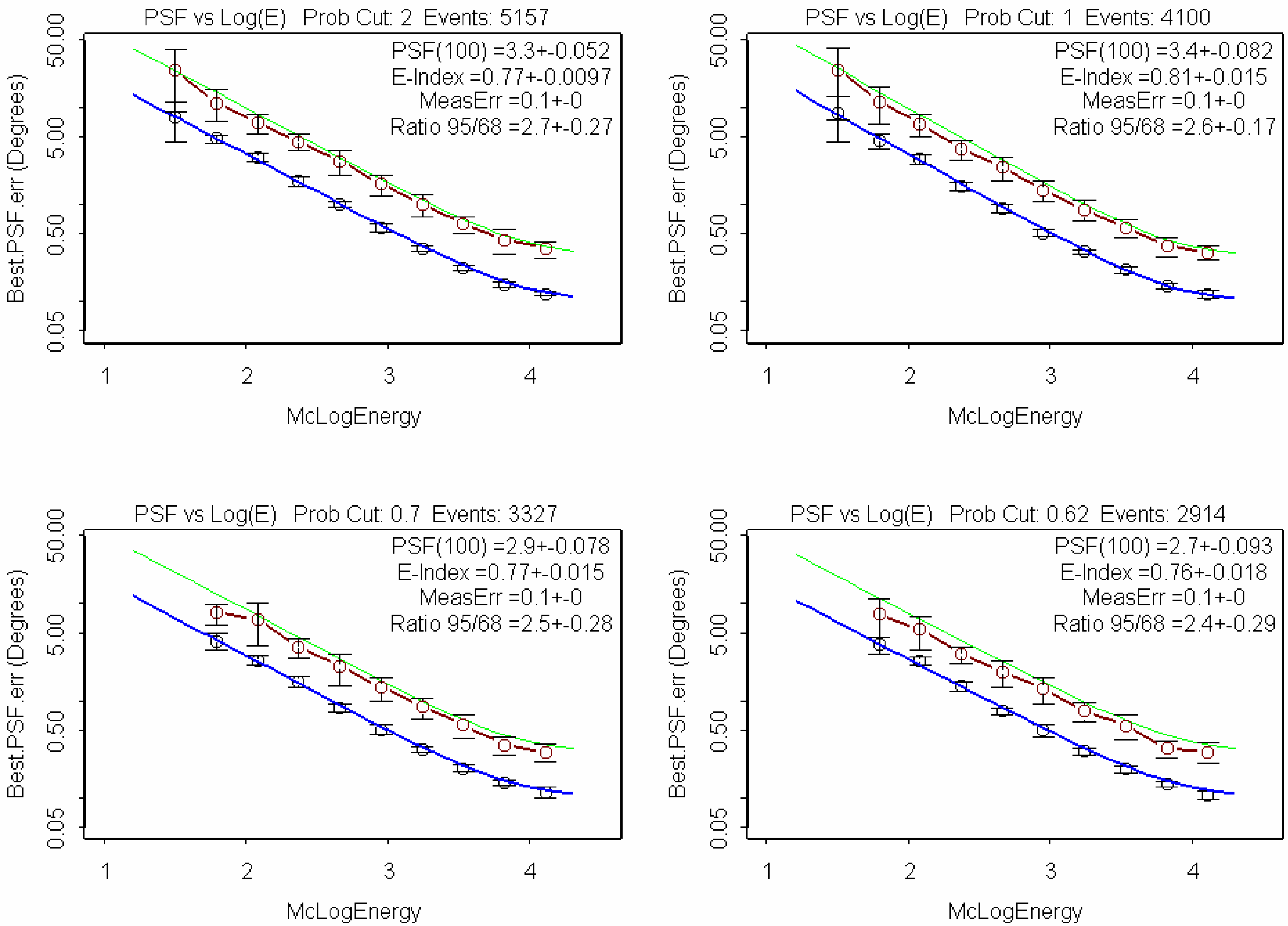
1) Analysis of covariance matrix gives format for modeling instrument response

2) Predictive power of Kalman Fit?

- Factor of 2.3

- May prove a good handle for CT tree determination of "Best PSF"

Present Analysis Status

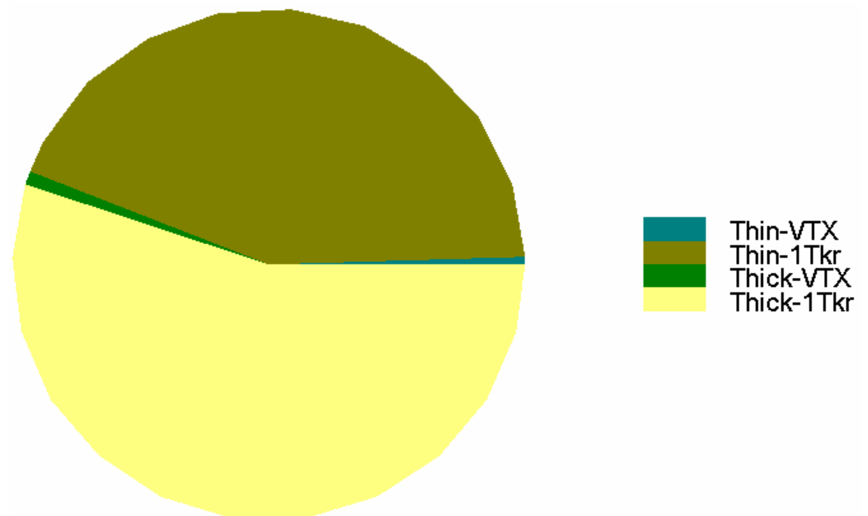


Present Status 2: BGE Rejection

Events left after
Good-Energy Selection: 1904

Events left in VTX Classes: 26

Events left in 1Tkr Classes: 1878



CT BGE Rejection factors obtained:

20:1 (1Tkr)

2:1 (VTX)

NEED FACTOR OF 10X EVENTS BEFORE PROGRESS CAN BE MADE!