MSC angular distributions in GEANT4 - where we are now ? (Draft)

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Abstract

Angular distributions of charged particles obtained with GEANT4 simulation are compared with experimental data and with the Highland parametrisation.

1 Introduction

We have simulated the multiple scattering of different charged particles in a broad energy range for several absorber materials with different thicknesses. The projected angular distribution is used in the Highland formula ([Hig75],[Hig79], [Lyn91]) - data - simulation comparisons. In these simulations GEANT4 version 5.1 was used in order to test the accuracy of the multiple scattering code. The setup used in the simulation was very simple, it was just a box of an absorber, the primary particles were electrons, protons, muons with a fixed energy. In most of the cases the central part of the angle distributions were tested because the Highland formula describe this part and at the same time there are only very limited number of experiments where (a part of) the tail was measured. In order to compare the simulation results with the data and/or the formula we have fitted the central part of the angle distribution with a Gaussian and we have compared the width with the measured/Highland values. The RMS width of the distribution is not a good choice to see the simulation - data differences because this parameter

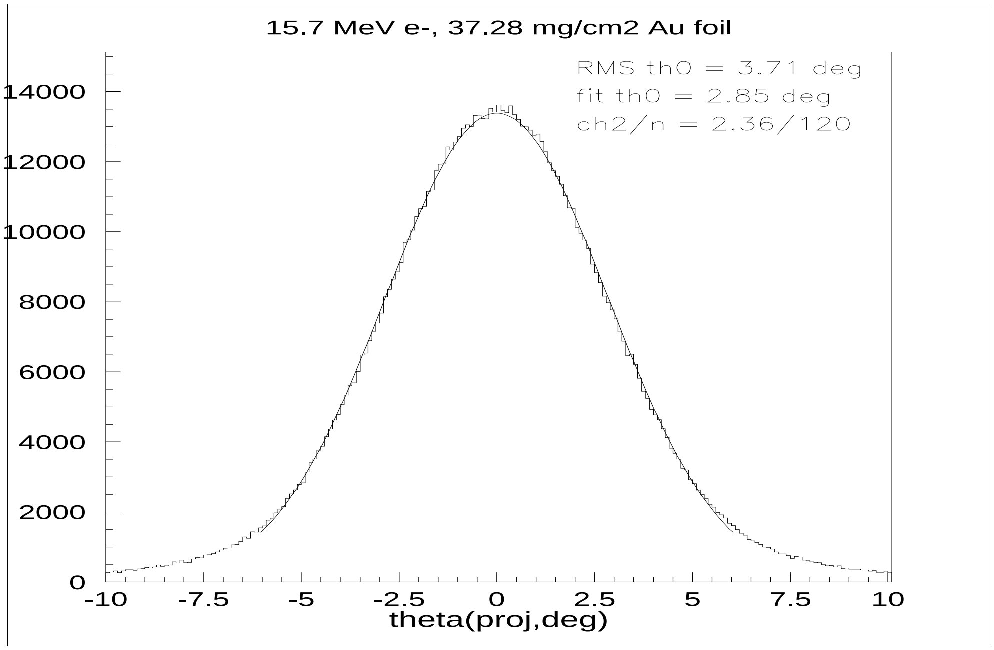

Figure 1: Fit of the angular distribution

depends on the tail as well.An example for this simple fittting procedure is shown in Fig.1, where there is a significant difference between the RMS and fitted widths.In this example the distribution was fitted in the [-6 , 6] interval with a Gaussian.

1.1 The Highland formula

This simple parametrisation of the central part of the distribution is

$$\theta_0 = \frac{13.6MeV}{\beta cp} z\sqrt{x/X0}(1 + 0.038 \ln x/X0) \tag{1}$$

where $\theta_0 = \theta^{rms}_{plane}$ is the width of the approximate Gauusian projected angle distribution, p,βc and z are the momentum, velocity and charge number of the incident particle, and $x/X0$ is the thickness of the scattering medium in radiation lengths. This value of θ_0 is from a fit to Molière distribution for singly charged particles with $\beta = 1$ for all Z, and is accurate to 11 % or better for $10^{-3} \leq x/X0 \leq 100$. (see e.g. Rev. of Particle Properties, section 23.3). The deviation of this θ_0 from the Molière value varies from material to material and depends on the thickness of the absorber, some examples can be found in [Lyn91]. Lynch and Dahl also present a better formula for θ_0 which is less simple than Eq. 1, but its claimed accuracy better, 2 %

for all Z. The Highland formula is easy to use, but if the energy loss of the projectile in the absorber is not small, the initial momentum of the particle is not good for using it in Eq.1. In this case an integration is needed to compute θ_0 properly, but there is a simple prescription to use the formula for big energy losses: take the geometric mean of initial and final kinematic factors i.e. $\beta cp = \sqrt{\beta_i cp_i \cdot \beta_f cp_f}$ see ([Gott93]).

2 Electron multiple scattering

2.1 Multiple scattering of 15.7 MeV electrons in thin gold foils

In their experiment ([Hans51]) A.O.Hanson et al. measured the angular distribution of the electrons after gold foils with thicknesses of 18.66 and 37.28 mg/cm2 (about 10 and 20 micrometers). Their experiment is one of the very few measurements where not only the small angle part of the distribution but the tail was measured too. They give the $1/e$ width of the small angle scattering part but they present the (spatial) angular distributions as well. This is the reason why we compare here the ditributions, not the widths only. In the simulation 5 micrometers production threshold was used ($\approx$ 50 keV in energy), the number of simulated events was 10^7 for each thickness. The data - simulation comparison can be seen in Figs.1. and 2.

In Fig.3. the G4 result is compared with the theoretical angle distribution. As it can be seen the simulation agrees well with the experiment and with the theory for this case.

2.2 Scattering of 2.25 MeV electrons

Kulchitsky and Latyshev [Kul42] measured the small angle region of the angle distribution for 2.25 MeV electrons in a set of materials. They fitted the measured distributions with a Gaussian and gave the values of the $\theta_{1/e}$ quantity (the angle where the distribution decreases by "e" times).

The production threshold used in the simulation was 1 mm.The number of events was $5 \cdot 10^5$ in every run in order to have a statistical error much smaller than the experimantal error, which can be estimated as 5%. For every simulated (projected) angle distribution a Gaussian was fitted with the form

$$\frac{dN}{d\theta} = Ce^{-\frac{(\theta - a)^2}{2\theta_0^2}} \tag{2}$$

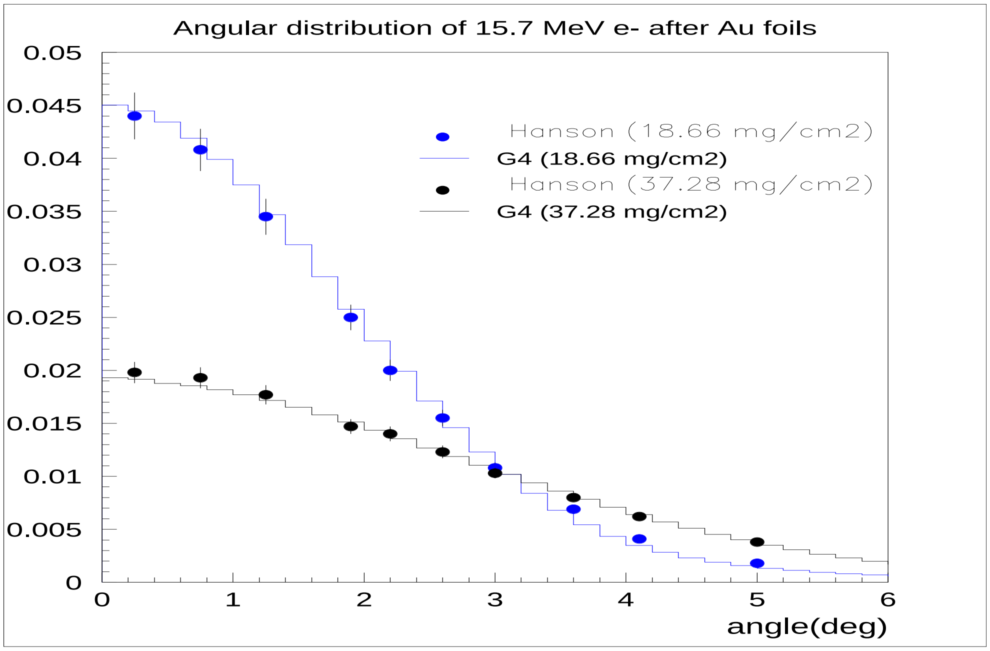

Figure 2: Angular distributions of 15.7 MeV electrons transmitted through gold foils, the data taken from [Hans51].

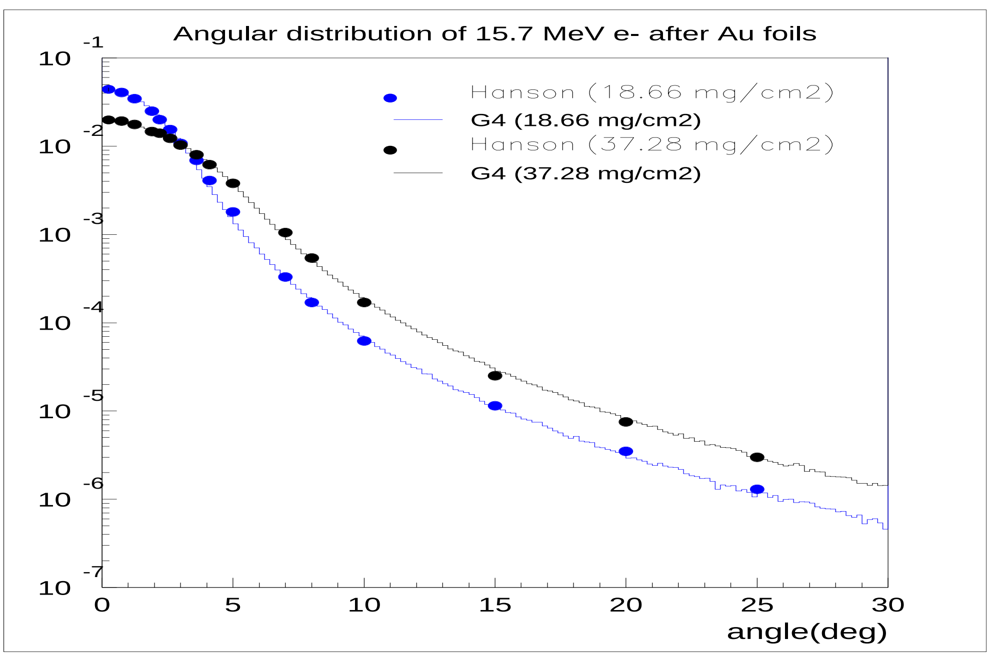

Figure 3: Angular distributions of 15.7 MeV electrons transmitted through gold foils, the data taken from [Hans51].

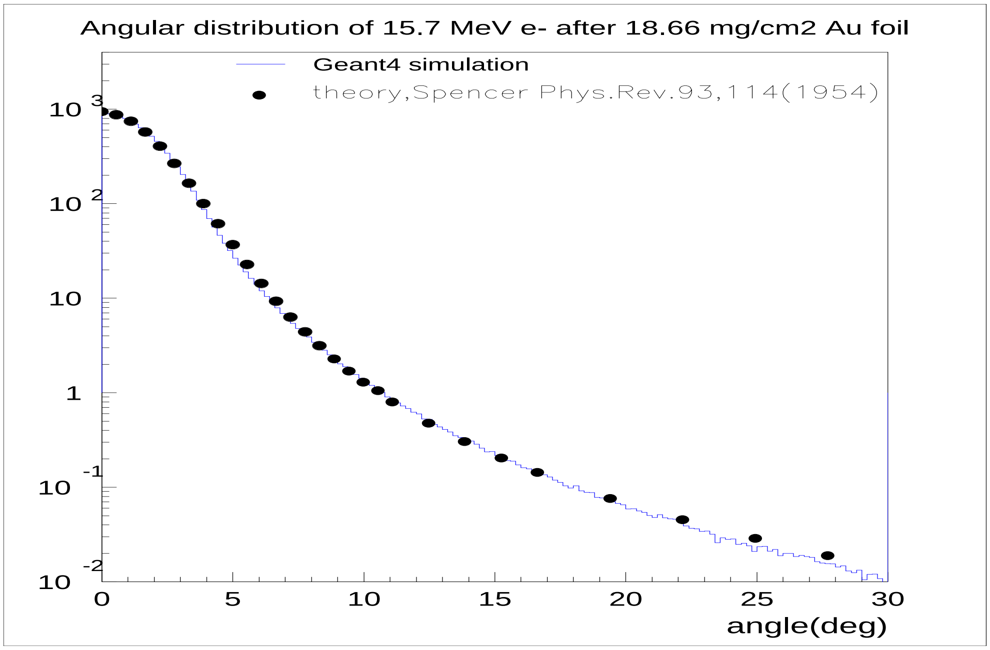

Figure 4: Angular distribution of 15.7 MeV electrons transmitted through a gold foil, G4 - theory comparison.

and then the computed width θ_0 was compared with the values from the experiment. (It is easy to see that $\theta_0 = \theta_{1/e}/sqrt(2)$.) The results are in Table 1, the table contains the theoretical values too, computed from the Goudsmit-Saunderson theory by Kulchitsky and Latyshev. Two of the measurements have been left out of the table, the data of the scattering on tin and gold have not been used. In the case of gold there are 2 different results in Table I and Table II of [Kul42], while for the case of tin absorber the thickness given in Table I is in error, it is not consistent with the measured value of $\theta_{1/e}$.

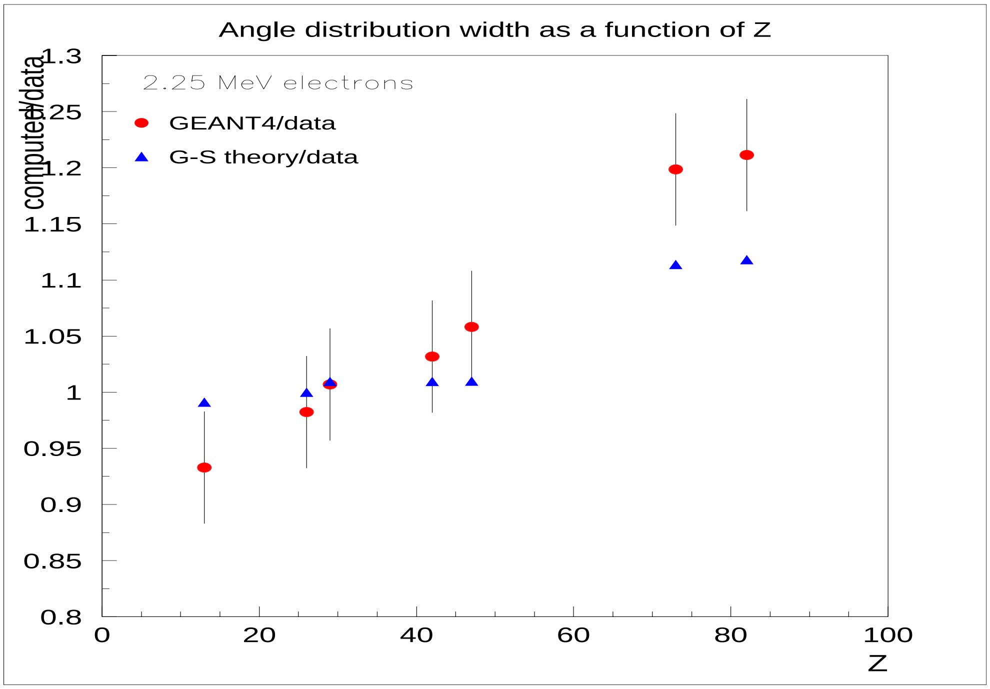

Figure 5: Angular distribution widths GEANT4 - theory - data comparison

Table 1

material	thickness mg/cm^2	θ_0^{G4} deg	θ_0^{H} deg	θ^{G-S} deg	θ_0^{data} deg
Al	26.60	6.26	7.27	6.65	6.71
Fe	15.40	6.67	7.27	6.79	6.79
Cu	17.15	7.40	8.03	7.42	7.35
Mo	12.40	7.48	7.78	7.32	7.25
Ag	11.55	7.63	7.87	7.28	7.21
Ta	8.90	8.33	7.93	7.74	6.95
Pb	7.90	8.31	7.70	7.67	6.86

As it can be seen from Table 1. GEANT4 gives the width rather close to the data from Al to Ag and it gives too big values for heavy elements, for Ta and Pb. The Highland parametrization gives too big values for all absorber, but for high Z it is closer to the data than GEANT4.It is interesting to note that the Goudsmit-Saunderson theory also predicts bigger θ_0 values for heavy elements, its results are bigger than the measured values by 10%. If we take into account the experimental errors (0.3-0.4 deg) together the behavior of the G-S theory, the GEANT4-data difference at high Z should not be considerd dramatic, we can say that GEANT4 nearly reproduces the data.

3 Proton scattering

3.1 Low energy protons

Bichsel measured the multiple scattering of low energy protons on different materials ([Bic58]). In his experiment the quantity $\theta_{1/e}$ was measured in Al,Ni,Ag and Au foils, the proton kinetic energy was in the range $0.766 - 4.76$ MeV. Several of his measurements was simulated in GEANT (with a production threshold of 1 mm, the number of events was $5 \cdot 10^5$ in every run). The comparison is contained in Table 2, where the simulated, Highland, theoretical and measured θ_0 values are compared.The error in the data has been estimated by Bichsel as $3 - 5\%$.

Table 2

material	Tp MeV	thickness mg/cm^2	θ_0^{G4} deg	θ_0^H deg	θ^M deg	θ_0^{data} deg
Al	0.766	1.37	3.05	3.04	3.38	3.14
Al	1.254	1.37	2.43	2.42	2.72	2.69
Al	2.355	3.42	1.43	1.43	1.60	1.68
Al	3.99	3.42	0.83	0.80	0.89	0.92
Al	4.76	3.42	0.70	0.66	0.74	0.78
Ni	1.20	2.18	3.40	3.25	3.37	3.37
Ag	2.02	12.4	7.60	7.73	7.35	7.35
Au	1.34	4.29	6.18	6.08	5.70	5.64
Au	2.02	4.29	3.92	3.81	3.61	3.86
Au	4.06	4.29	1.88	1.78	1.70	1.80

It can be seen from Table 2 that GEANT4 5.1 reproduces the data and Molière results quite well. The Highland formula also gives good results in this case despite the fact that the normalized target thicknesses $x/X0$ are smaller than the accepted lower limit of the validity 0.001 ($6 \cdot 10^{-5} \leq x/X0 \leq 6 \cdot 10^{-5}$).

3.2 Protons of medium energy

B.Gottschalk et. al. ([Gott93]) have measured multiple scattering of 158.6 MeV protons in a number of materials from beryllium to uranium. Their targets ranged from very thin (negligible energy loss) to very thick (grater than the mean proton range). Each of the data sets was fitted with a Moliere scattering distribution to extract the characteristic angle θ_M and a Gaussian distribution to get the characteristic angle θ_0. We have simulated a part of their measurements and the results are presented ina Table 3, where the θ_0 widths are compared from GEANT4, from the Highland formula and from the experiment. Table 3 also gives the mean energy loss of the proton in the absorber in the last column.

Table 3

material	thickness g/cm^2	θ_0^{G4} mrad	θ_0^{H} mrad	θ_0^{data} mrad	error mrad	en.loss MeV
Be	0.0572	1.088	1.003	0.993	0.057	0.24
Be	0.6440	3.942	3.824	3.743	0.224	2.76
Be	4.7580	11.925	12.042	11.821	0.250	21.44
Be	18.5060	38.767	40.695	34.086	0.346	109.12
Be	20.5270	51.572	63.463	43.979	0.680	136.49
Al	0.2160	3.709	3.599	3.534	0.491	0.89
Al	2.1729	13.041	12.989	13.104	0.202	8.92
Al	11.9570	39.812	38.982	39.986	0.582	56.62
Al	21.9150	100.49	135.52	88.657	2.110	144.99
Al	22.3300	106.38	171.88	94.390	10.271	155.45
Pb	0.0290	2.537	2.480	2.304	0.172	0.08
Pb	3.5080	34.018	34.519	35.029	0.325	9.60
Pb	20.1960	109.160	108.420	107.092	1.021	62.12
Pb	34.3460	239.010	323.05	204.530	10.099	141.92
Pb	35.2600	239.660	425.520	201.132	14.818	153.86

For this proton energy range GEANT4 agrees well with the data except the case of very thick absorbers (where the particle range $\approx$ thickness), here GEANT4 overestimate the scattering angle by $\approx 10-20$ %. The Highland formula gives similar results, but for thick targets it gives a result too big by $\approx 60-100$ % as it can be seen in Fig. 6.

3.3 High energy protons

G.Shen et al. ([Shen79]) studied the multiple scattering of high energy hadrons. They measured the $1/e$ width of the angular distribution for protons,antiprotons,pions and kaons with various targets from hydrogen to lead in the momentum range $50-200\ GeV/c$ and compared the results with the Molière theory and with the Highland formula as well. Table 4. gives the G4 simulation - Moliére theory - Highland - data comparisons for the θ_0 widths.The primary particle was proton in all of the simulation runs, we used 1 mm production threshold and genarated $5\cdot 10^5$ events by run.

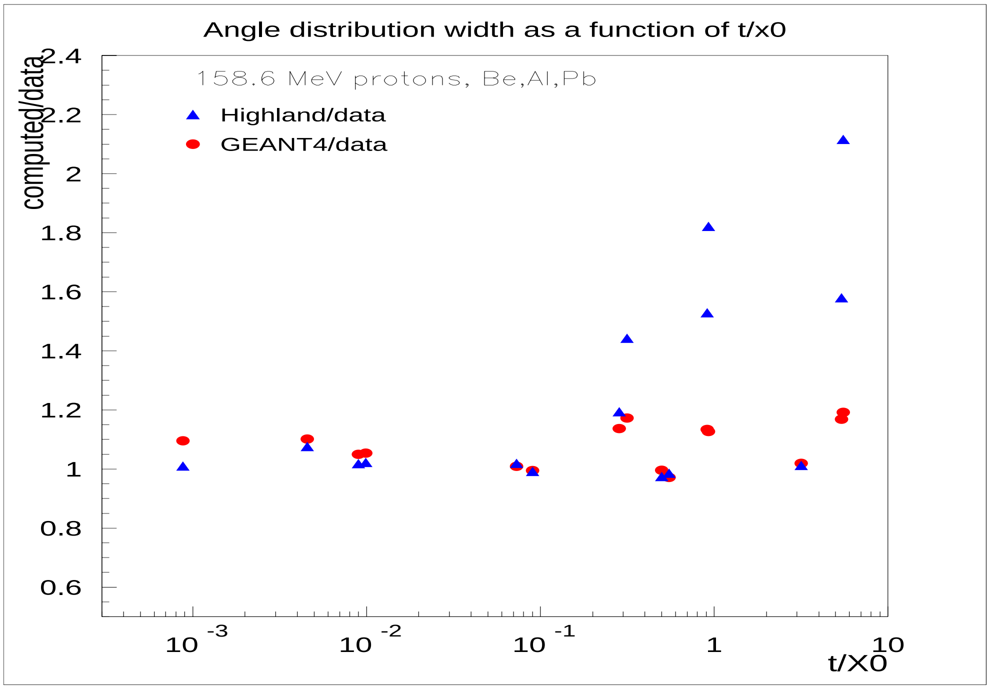

Figure 6: Angular distribution widths Highland - GEANT4 - data comparison

Table 4

material ≈ 0.1 r.l.	momentum GeV/c	θ_0^{G4} 10^{-5} rad	θ_0^{M} 10^{-5} rad	θ^{H} 10^{-5} rad	θ_0^{data} 10^{-5} rad	exp.error 10^{-5} rad
H	50.	6.95	7.72	8.58	7.70	0.14
H	200.	1.72	1.93	2.14	1.81	0.23
Be	70.	4.55	4.98	5.31	4.93	0.59
Be	125.	2.45	2.79	2.99	1.87	0.98
Be	175.	1.83	1.99	2.12	2.61	0.44
Al	70.	4.70	5.07	5.30	5.17	0.35
Al	125.	2.43	2.84	2.97	2.78	0.57
Al	175.	1.72	2.03	2.11	1.93	0.39
Pb	70.	4.19	5.27	5.33	5.53	0.32
Pb	125.	2.33	2.94	2.98	2.65	0.53
Pb	175.	2.13	2.11	2.12	2.13	0.29

The Be,Al and Pb results are plotted in Fig. 7 as well.From the figure and Table 4 we can conclude that GEANT4 v5.1 tends to underestimate the width for high energy hadrons, although -looking at the rather big experimental

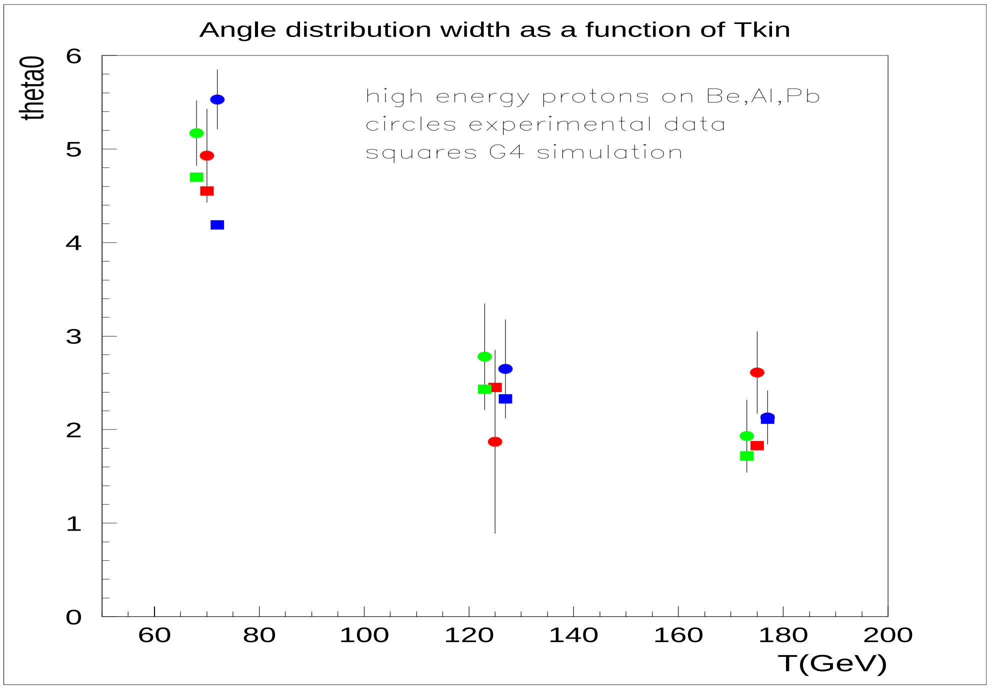

Figure 7: Angular distribution widths for high energy protons, GEANT4 - data comparison

errors- the difference is not very large here.

4 Muons

I have found only one measurement in the literarture for the measurement of muon multiple scattering, where the energy of the muons was known ([Lag74]). For this low energy muons ($9 - 25$ MeV) GEANT4 v5.1 agrees well with the measurements, it gives the same widths than the experiment. For muons of high energy there are no experimental data available so only a Highland - GEANT4 comparison has been made. A thin target case and a thick taget case have been simulated, the first is 1 GeV muons with 0.03 X0 tungsten, the second is 100 GeV muons with 1 meter iron, the projected angle distributions are plotted in Fig.8 and Fig.9.

Geant4 gives in both cases too narrow distributions compared to the Highland parametrization. Geant4 θ_0 is smaller by ≈ 16 % for thin and ≈ 30 % for thick absorber.

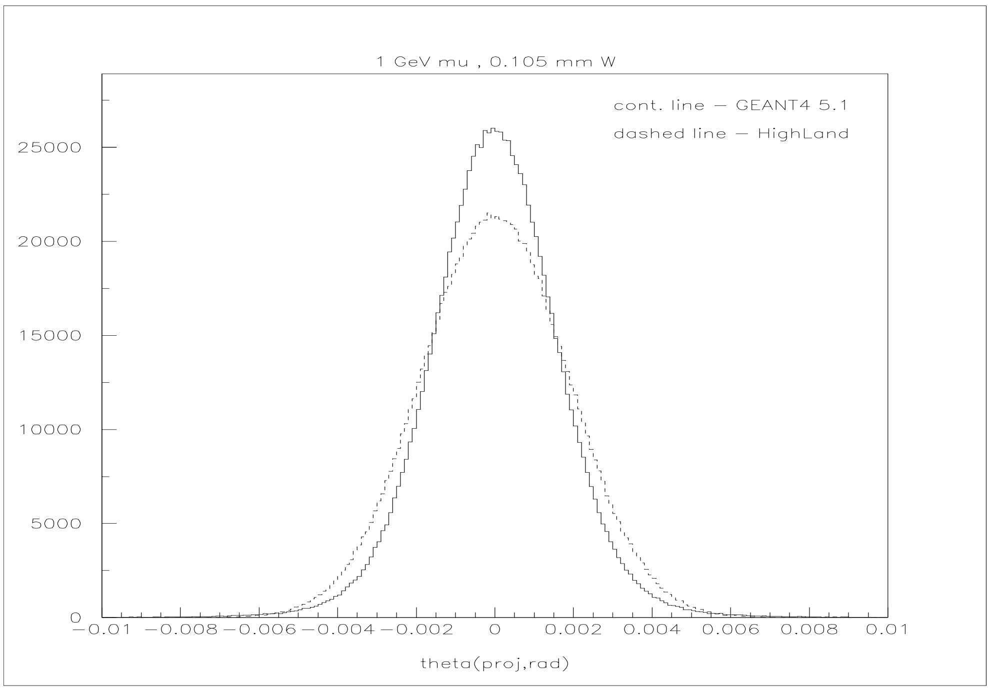

Figure 8: Angular distribution widths for 1 GeV muons in 0.105 mm W, GEANT4 - Highland comparison

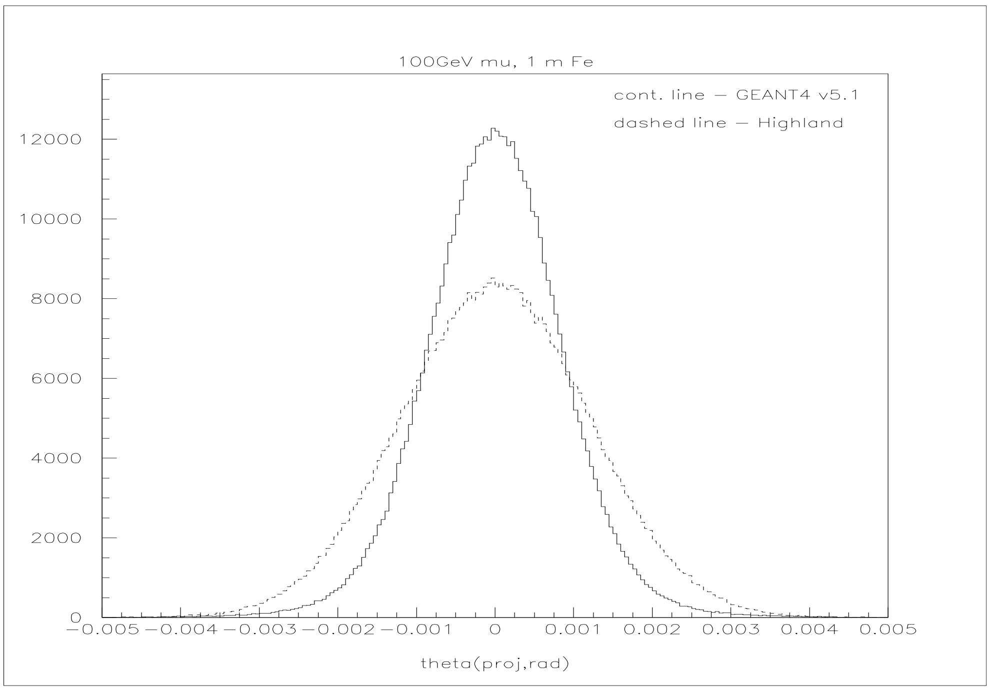

Figure 9: Angular distribution widths for 100 GeV muons in 1 m Fe, GEANT4 - Highland comparison

5 Conclusions

From the results given above we have concluded the the multiple scattering code in GEANT4 v.5.1 produces good results for electrons and for heavy particles of low and moderate energy, but it tends to underestimate the scattering for high energy muons and hadrons. Therefore a modification has been made in the code which will go to version 5.2.

The main change in the code that now the model parameter which detemines the width of the central part of the angle distribution is computed according to the Highland formula (for all particles but e-/e+) and this version of course is agreement with the Highland parametrization.There is no significant change for electrons/positrons. However some modification may be necessary for electrons as well, studies are going on. (The correction has been applied for muons/hadrons is not a good one in this case, because applying it the tail of the 15.7 MeV e- angle distribution becomes worse.) The most important thing for the further development/tuning of the multiple scattering code in GEANT4 is to have acces to some data of good quality, in this every help is welcome.

6 Appendix

Here some results are given using the new GEANT4 version and GEANT3 (with the same conditions,cuts,..). Fig. 10 shows the GEANT4 - GEANT3 - data comparison for 2.25 MeV electrons scattered on different targets.As it can be seen GEANT4 $\approx$ GEANT3 $\approx$ data for small/medium atomic numbers, while GEANT3 gives results closer to the data for high Z.

Table 5 shows the results for multiple scattering of low energy protons. Here G4 v5.2 is compared with G3, the Molière calculation and with the data measured by Bichsel. We can see in the Table that the G4 results are quite close to the data/theory, but G3 gives an angle distribution which is too wide in a number of cases. The G3 result comes from the fact that GEANT3 tends to overestimate the energy loss for low energy particles, like the protons here.

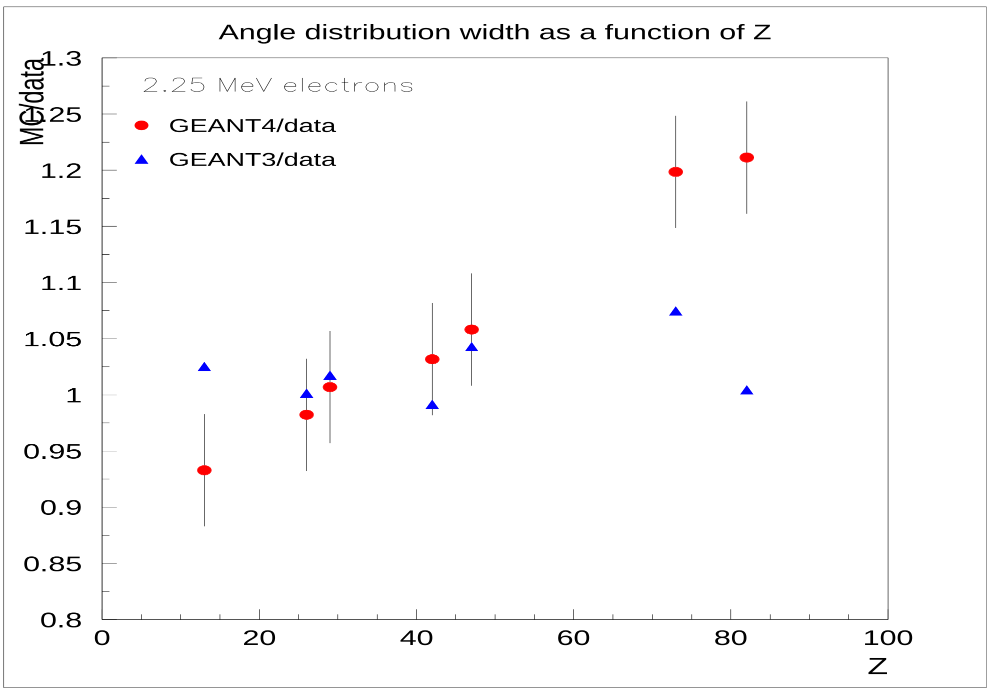

Figure 10: Angular distribution widths GEANT4 - GEANT3 - data comparison, (for electrons GEANT4 v5.2 = v5.1)

Table 5
Multiple scattering of low energy protons
(Geant4 v5.2 - GEANT3 - Molière - data)

material	Tp MeV	thickness mg/cm^2	$\theta_0^{G45.2}$ deg	θ_0^{G3} deg	θ^M deg	θ_0^{data} deg
Al	0.766	1.37	3.30	4.44	3.38	3.14
Al	1.254	1.37	2.59	3.43	2.72	2.69
Al	2.355	3.42	1.48	1.82	1.60	1.68
Al	3.99	3.42	0.82	0.94	0.89	0.92
Al	4.76	3.42	0.67	0.76	0.74	0.78
Ni	1.20	2.18	3.46	3.85	3.37	3.37
Ag	2.02	12.4	7.98	9.98	7.35	7.35
Au	1.34	4.29	6.33	6.05	5.70	5.64
Au	2.02	4.29	3.95	3.67	3.61	3.86
Au	4.06	4.29	1.82	1.64	1.70	1.80

Fig.11 gives results for the scattering of 158.6 MeV protons on a number of target with different thicknesses. As it can be seen in the Figure both GEANT4 and GEANT3 gives good results for thin targets ($thickess \leq 0.5 \cdot X_0$) but for thick absorbers GEANT4 gives better angular distribution widths.

Table 6 is a short summary of the simulation of high energy protons. It shows that in this case GEANT4 v5.2 and GEANT3 gives similar results, both simulations agree with the data within errors except the scattering on hydrogen target, where G4/G3 gives bigger scattering than that is seen in the experiment.

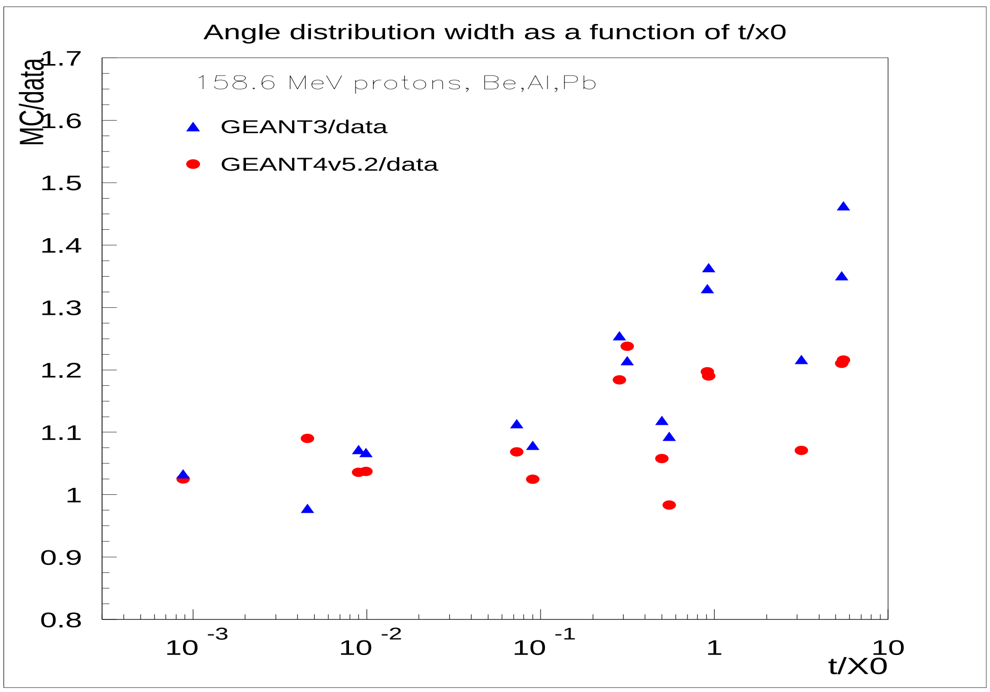

Figure 11: Angular distribution widths for 158.6 MeV protons, GEANT3 - GEANT4 v5.2 - data comparison

Table 6
Multiple scattering of high energy protons
(GEANT4 v5.2 - Molière - GEANT3 - data comparison)

material ≈ 0.1 r.l.	momentum GeV/c	$\theta_0^{G4v5.2}$ 10^{-5} rad	θ_0^M 10^{-5} rad	θ^{G3} 10^{-5} rad	θ_0^{data} 10^{-5} rad	exp.error 10^{-5} rad
H	50.	9.32	7.72	8.90	7.70	0.14
H	200.	2.37	1.93	2.24	1.81	0.23
Be	70.	5.59	4.98	5.85	4.93	0.59
Be	125.	3.07	2.79	3.21	1.87	0.98
Be	175.	2.23	1.99	2.35	2.61	0.44
Al	70.	5.42	5.07	5.42	5.17	0.35
Al	125.	3.07	2.84	3.01	2.78	0.57
Al	175.	2.18	2.03	2.14	1.93	0.39
Pb	70.	5.44	5.27	5.49	5.53	0.32
Pb	125.	3.03	2.94	3.07	2.65	0.53
Pb	175.	2.17	2.11	2.17	2.13	0.29

For high energy muons the GEANT4 v5.2, GEANT3 results identical with the angle distribution widths given by the Highland formula 1.

7 Acknowledgement

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