Likelihood Performance

Likelihood Tool
Performance

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Statistical Models

- Instrument response

\[
R(E', \hat{p}'; E', \hat{p}, t) = A(E, \hat{p}, t)P(\hat{p}'; E, \hat{p}, t)D(E'; E, \hat{p}, t).
\]  

\[E, \hat{p} = \text{true photon energy and momentum}; \ E', \hat{p}' = \text{measured energy and momentum}.
\]

- Unbinned likelihood

\[
\log L = \sum_j \log M(E'_j, \hat{p}'_j, t_j) - N_{\text{pred}}
\]  

where

\[
M(E', \hat{p}', t) = \sum_i \left[ \int_{\text{SR}} dE d\hat{p} R(E', \hat{p}', t; E, \hat{p}) S_i(E, \hat{p}') \right],
\]

\[
N_{\text{pred}} = \int_{\text{ROI}} dE' d\hat{p}' dt M(E', \hat{p}', t)
\]

Sources are indexed by \(i\); events are indexed by \(j\). \(\text{SR}\) is the “source region”; \(\text{ROI}\) is the “region-of-interest”.
Binned likelihood

\[ \mathcal{L} = \prod_j \frac{\theta_{n_j}^j e^{-\theta_j}}{n_j!} \quad (5) \]

where

\[ \theta_{ij} = \int_j dE' \, d\hat{p}' \int dt \int_{SR} dE \, d\hat{p} \, R(E', \hat{p}'; E, \hat{p}, t) S_i(E, \hat{p}), \quad (6) \]

\[ \theta_j = \sum_i \theta_{ij}. \quad (7) \]

\( n_j \) is the number of events in pixel \( j \).

Current implementation neglects energy dispersion, i.e.,

\[ D(E'; E, \hat{p}, t) \equiv \delta(E - E') \quad (8) \]
**Tests**

- **Basic Procedure**
  - Generate data with `gtobssim` using specific set of IRFs (DC1, GLAST25, testIrfs) and default step-rocking orbit/attitude calculation provided by flux package.
  - Fit the resulting data with Likelihood tool using its parallel implementation of the model, e.g., use the same FITS image template as for the simulation.
  - Similar steps could be used for assessing source detection limits, expected constraints on source models, etc., i.e., this procedure is the cognate of `xspec/fakeit` feasibility studies.

- **Fit Accuracy I**
  Perform many simulations/fits, 1 day observation time, plot distributions of fit parameters, look for biases wrt MC truth.
  1. Single source, power-law spectrum, no diffuse emission
  2. Three sources, comparable fluxes, differing power-laws, no diffuse
  3. Extragalactic diffuse (isotropic), power-law spectrum
  4. Galactic diffuse (EGRET model), power-law spectrum
• **Fit Accuracy II**
  - Longer observation (1 week), 3EG sources, with and without EG and Galactic diffuse.
  - Fit for many sources ($\gtrsim 20$)
  - Consider various fields – high and low Galactic latitude
  - Plot fit parameters (flux, spectral index) vs MC truth

• **Error Estimates**
  Compare estimates from covariance matrix to projected posterior distributions from MCMC.

• **Execution Times**
  - Linear scaling with number of events for unbinned and number of pixels for binned analysis.
  - Linear scaling with number of fit parameters.
  - Startup costs.
Unbinned fit to a single point source: Crab Pulsar

Crab_Pulsar

Entries / bin

Prefactor

Crab_Pulsar

Entries / bin

Index
Binned fit to Crab Pulsar

Crab_Pulsar

Entries / bin

Prefactor

Crab_Pulsar

Entries / bin

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24 28 32

-2.3 -2.2 -2.1

-2.3 -2.2 -2.1

0 5 10 15 20

0 5 10 15 20
Simultaneous fit to Crab, Geminga, PKS0528+134
Unbinned fit to Crab, Geminga, PKS0528+134
Binned fit to Crab, Geminga, PKS0528+134
Binned fit to Isotropic Diffuse

EG_Diffuse

Entries / bin

Prefactor

0 1.45 1.50 1.55 1.60 1.65 1.70

EG_Diffuse

Entries / bin

Index

-2.15 -2.10 -2.05
Binned fit to EGRET Galactic Diffuse Model

GalDiffuse

Entries / bin

Prefactor

GalDiffuse

Entries / bin

Index
Binned fit of Galactic Center region
Binned fit of Galactic Center region

35 point sources, no diffuse
Binned fit of Galactic Center region

15 brightest point sources, Galactic and Extragalactic diffuse
**Error estimate reliability**

- Marginalized posterior distribution from MCMC (histogram), Gaussian function representing best-fit and estimated 1-sigma errors (red curve) for a single point source
- For three bright sources fit simultaneously
Binned fit execution times

![Graph showing the relationship between the number of point sources and CPU time. The graph is a linear plot with points connected by a straight line, indicating a direct proportionality between the number of point sources and the CPU time.]