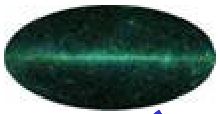
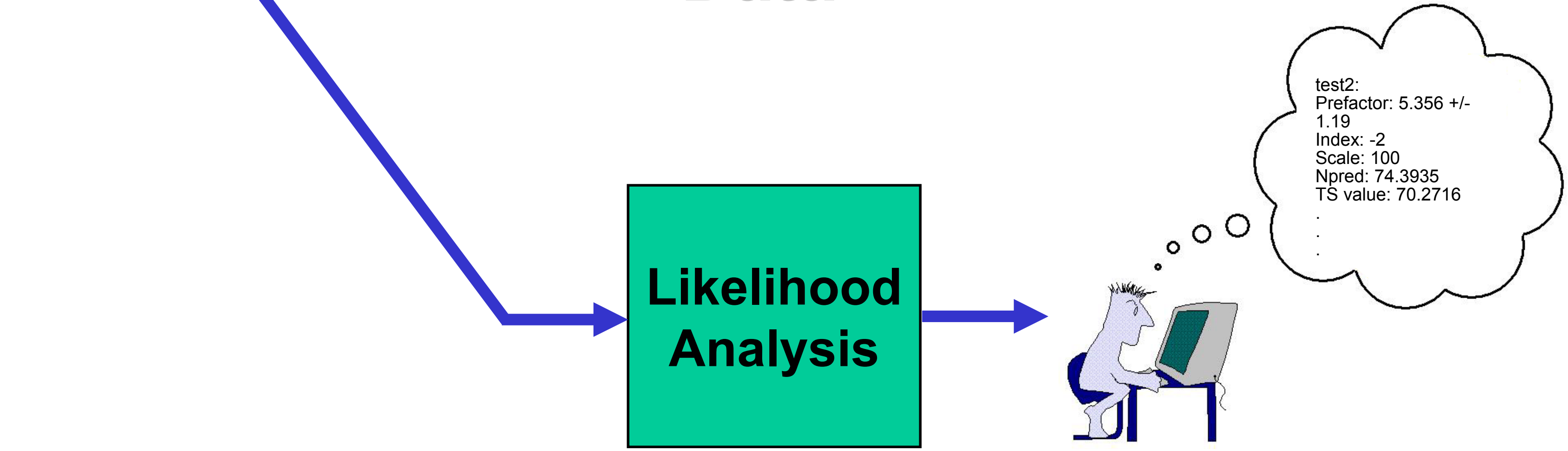


Statistical Issues in Likelihood Analysis of LAT Data



(Apologies to R. Dubois, D. Flath)

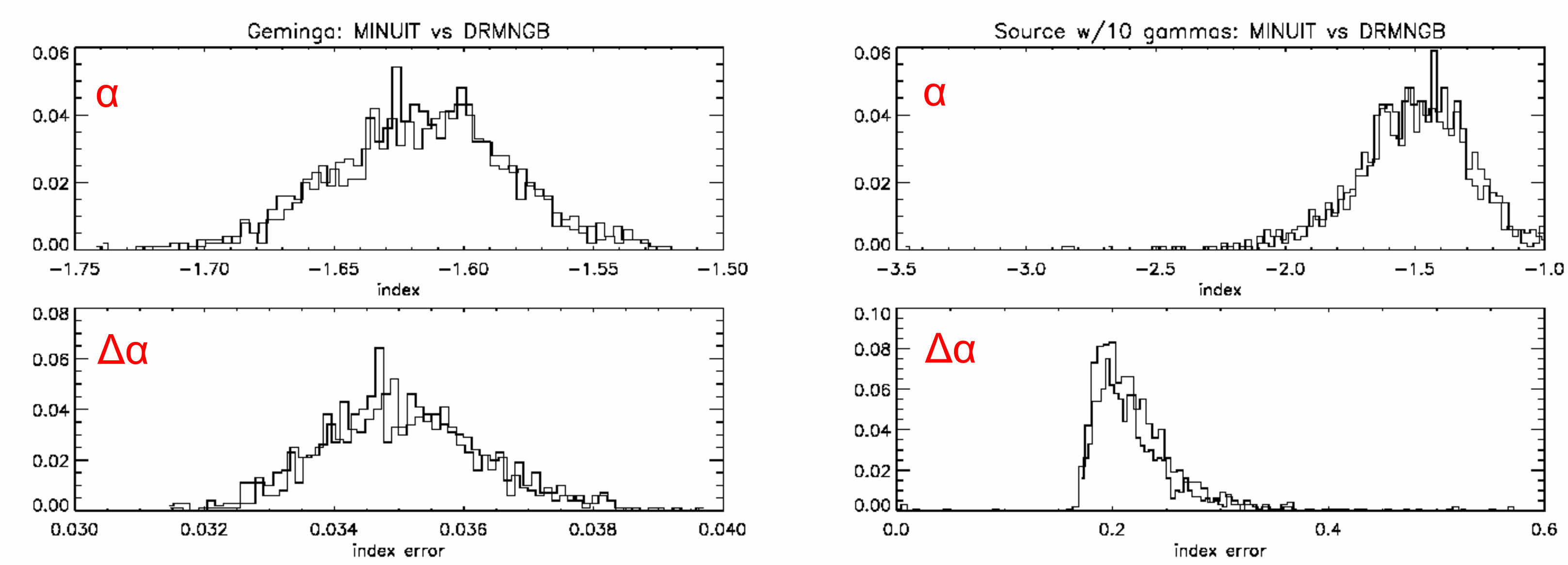
Seth Digel (HEPL/Stanford Univ.) & Guillaume Dubus (LLR/IN2P3)

Introduction

- **Why investigate the output of likelihood analysis using known inputs?**
  - Verify that the analysis is working (biases in parameter determinations, correctness of confidence ranges, for example)
  - Infer the performance of the LAT for a particular circumstance (a given pointing history and source model)
- **Here we present some initial results using likelihoodApp.exe, TsMap.exe, and obsSim.exe**
  - See also Guillaume's posted report

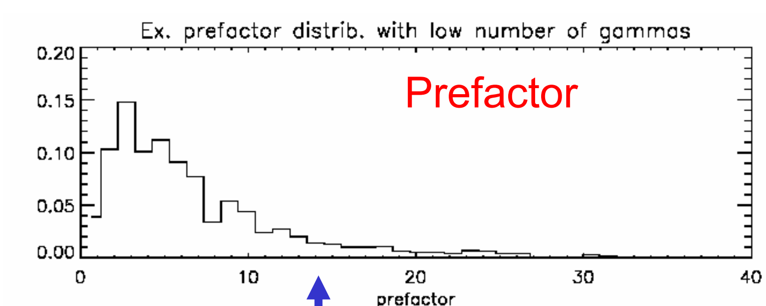
likelihoodApp.exe Optimizers

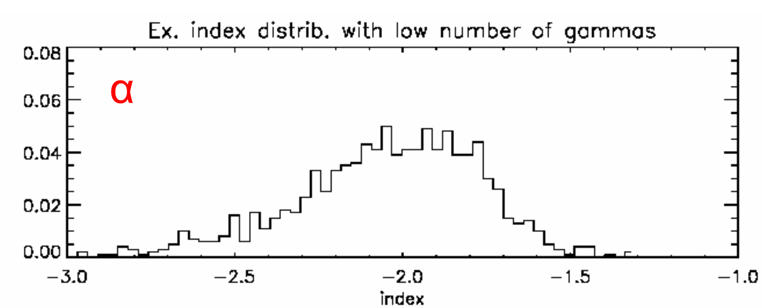
- 3 optimizers are offered
- Results of MINUIT and DRMNGB are equivalent, in the simple optimization cases (no diffuse emission) investigated (bright and faint sources)



Confidence Regions

- **likelihoodApp confidence ranges are the ‘square root of the covariance matrix’, which under assumptions that we’d like to make corresponds to the 68% confidence interval**
- **Fits with no diffuse emission indicate that the confidence intervals of the maximum likelihood spectral index are 68%, or maybe a little more conservative for low-count sources, but the reported intervals for the prefactor can be underestimates**
- **More work is needed**

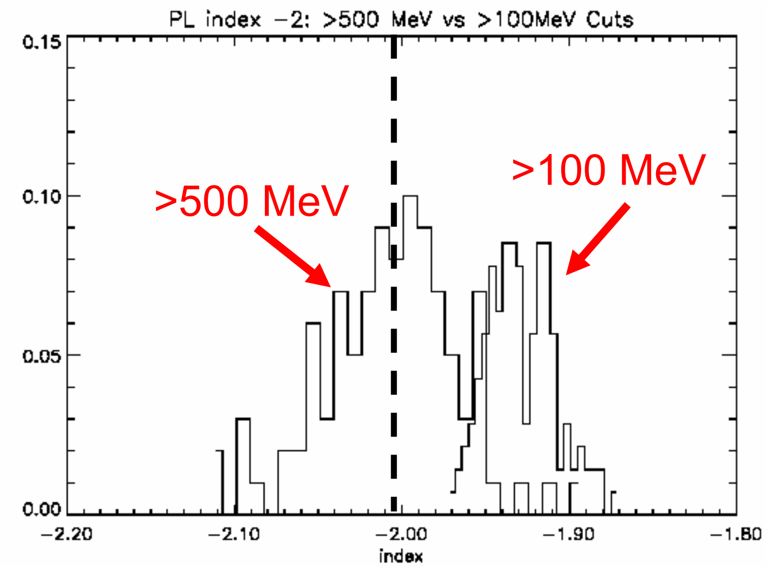




Note that Prefactor cannot be negative; the distribution cannot be Gaussian (especially apparent for this faint source)

Biases in Parameter Estimation

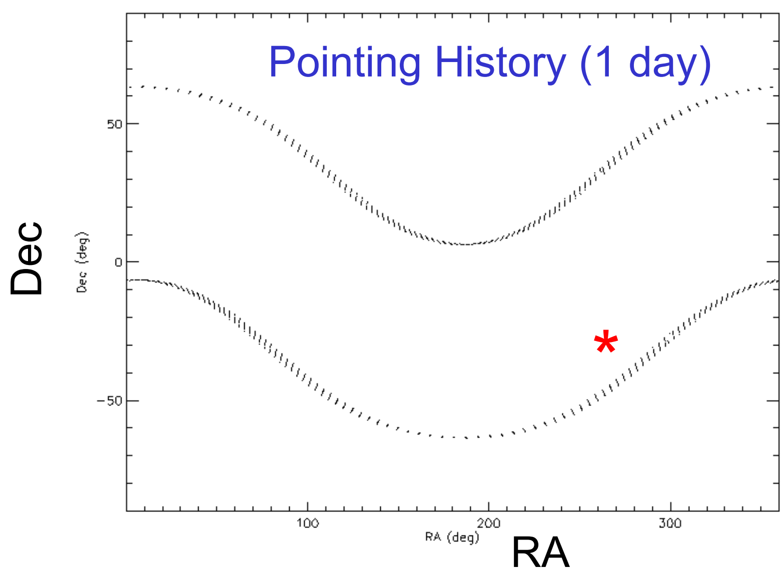
- No systematic investigation yet, but at least for spectral index fitting, a bias is evident
- Especially so if lower-energy events are included
  - Important note: in this simulation, Emin was 100 MeV
- obsSim.exe uses the energy redistribution functions (and so does Gleam, effectively), but likelihoodApp.exe does not (yet)
- So the loss (dispersion below Emin) of gamma-rays at low energies results in an apparent hardening of the spectrum



100 trials, true spectral index -2

Investigations of Source Detection with likelihoodApp.exe

- Tests with one day's worth of exposure, DC1 style
- Phony source at Galactic center
  - E^{-2} photon number spectrum, no break, flux (>100 MeV) 5 x 10^{-7} cm^{-2} s^{-1} [~typical fairly bright for EGRET source]
- Isotropic background, 1.5 x 10^{-5} cm^{-2} s^{-1} sr^{-1}, -2.1 spectral index
- The GC is not representative of the typical direction on the sky in terms of coverage by the LAT during this day
  - Somewhat better than average in terms of coverage



Test Statistic Maps

- See Mattox et al. (1996) for EGRET usage

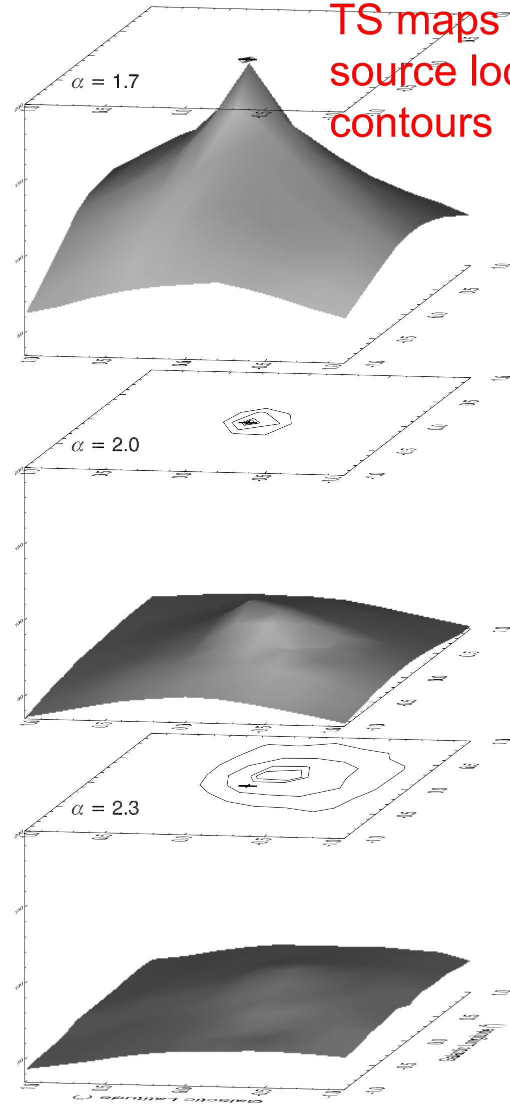
$$T_s \equiv -2(\ln L_0 - \ln L_1)$$

- Searches for point sources were implemented as brute force comparisons of models with a trial additional point source, tested at each point of a grid. For each grid point, the values of all parameters were reoptimized (i.e., the likelihood function was maximized)
- The significance of the resulting improvement of the likelihood (the value of TS) was interpreted quantitatively in terms of the χ^2 distribution with the number of d.o.f. equal to the difference in number of free parameters between the models, with appeal to Wilk's Theorem
  - This is strictly speaking, not a valid application of Wilk's theorem (e.g., Protassov et al. 2002), and needs to be verified through simulation
- For source location determinations, contours of ΔTS around the peak position are used to define confidence ranges (χ^2 with 2 deg of freedom, so, e.g., the 99%, 95%, 68%, and 50% contours are 9.2,6.,2.3, and 1.4 below the peak)

Effect of Spectral Index and Spectral Cutoffs

- **Range of photon spectral indicies for 3EG sources is approximately 1.5-3.5, although most are close to 2 (and the spectral index is poorly determined for many sources)**
  - **Pulsar spectra tend to be hard, but to roll off in the ~1 GeV range**
- **Influence on determination of source location is dramatic**

TS maps with source location contours



10-7 cm-2 s-1 (>100 MeV, α = -2), 1.5x10-5 cm-2 s-1 sr-1 (>100 MeV, α = -2.1) background

α	E_{max} (GeV)	Diameter 95% confidence contour
1.7	100	~3
2.0	100	5.9
2.3	100	10
1.7	1	11
2.0	1	18
2.3	1	24

Where the TS Comes From

- The contributions to the TS from different energy ranges or event types can be tallied separately
- For now, a cheat was employed, using likelihood analysis for separate energy ranges (instead of a single model for the entire energy range), although a fixed α was used

5×10^{-7} cm^{-2} s^{-1} (>100 MeV, α = -2)
1.5×10^{-7} cm^{-2} s^{-1} sr^{-1} (>100 MeV, α = -2.1) background

E_{min} (MeV)	E_{max} (MeV)	N_γ	Prefactor*	TS
30	1e5	1110	5.54 ± 0.84	128
30	100	322	11.3 ± 2.9	30
100	300	422	4.56 ± 1.27	25
300	1e3	264	6.16 ± 1.52	61
1e3	3e3	72	4.69 ± 2.25	15
3e3	1e5	30	1.29 ± 1.93	1
30	1e5	576	4.58 ± 0.98	69
30	1e5	534	7.23 ± 1.51	61

~'Sweet spot' → (row 300–1e3)

FRONT-only → (row 30–1e5, 576)

BACK-only → (row 30–1e5, 534)

$TS^{1/2}$?

*Correct answer: 5

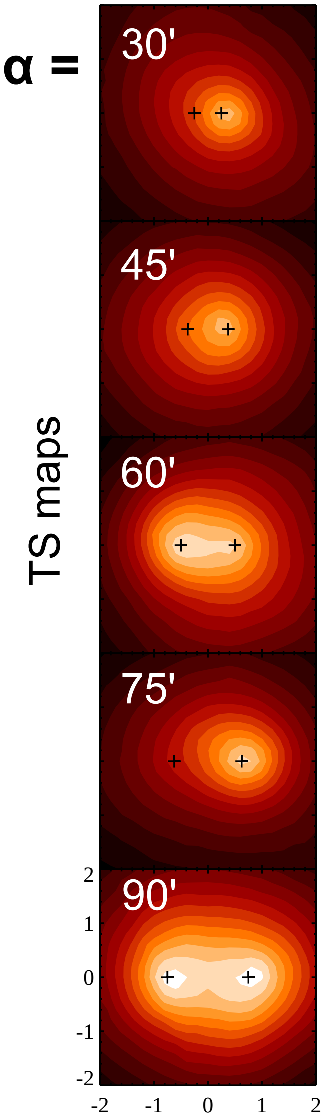
Resolving Closely-Spaced Sources

- **Sources each have flux 10^{-7} cm^{-2} s^{-1} (>100 MeV), α = -2**
- **Background 1.5 x 10^{-5} cm^{-2} s^{-1} sr^{-1} (α = -2.1)**
- **Analysis for energies 30 MeV – 100 GeV**
- **Only one trial for each source separation**

Separation	TS*
15'	8.0
30	50
45	24
60	71
75	100
90	90
120	92

← Convergence problem?

*Interpret as 2 source vs. 1 source test with only 1 dof difference



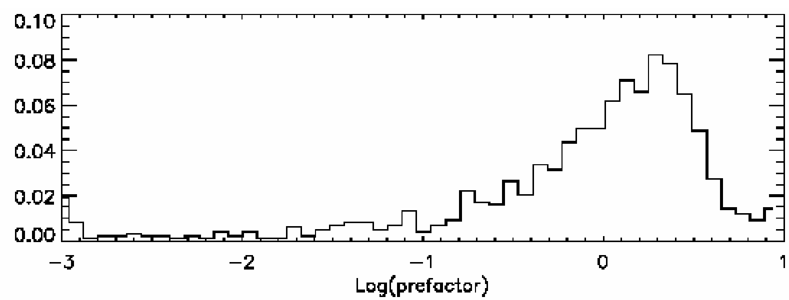
Toward the Flux Limit

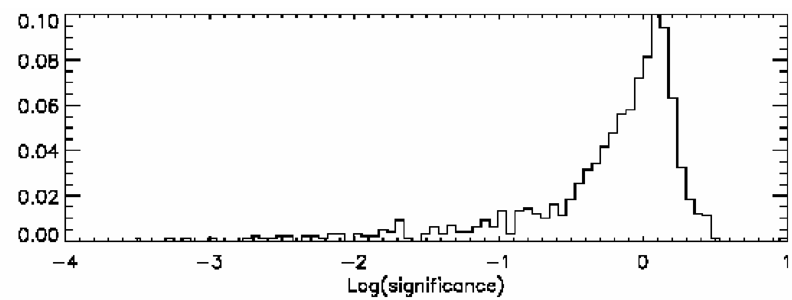
- Same setup as usual, and again only one trial per flux

Flux (10^{-7} cm^{-2} s^{-1}, >100 MeV)	TS*
1.0	4.0
1.5	12
2.0	18
2.5	66
5.0	128

Spurious Source Rate

- Fitting a point source where there is only diffuse emission in the data
- Only initial results (significance is Prefactor/[uncertainty of prefactor])





Conclusions

- The DC1 science tools provide the means to test our assumptions about the statistical interpretation of the likelihood analysis
- Initial investigations suggest that, e.g., confidence ranges are accurate
- With the likelihood tool, source localization behaves in an expected way with source spectrum, event type
- The flux limits, resolution limits, etc., can be inferred from likelihood analyses of the simulated data