GLAST Energy

*or* Humpty-Dumpty’s Revenge

- A Statement of the Problem
- Divide and Conquer strategy
- Easiest: Leakage (Depth) Correction
- Next Hardest: Tracker Sampling (Details Matter)
- Hardest: Edges (Tracker & Calorimeter)
- Present status
GLAST’s Fracture Energy

1 GeV $\gamma$

Thin Radiator Hits

Thick Radiator Hits

Blank Radiator Hits

Calorimeter Xtals

Gap Between Tracker Towers

Gap Between CAL. Towers

Leakage out CAL. Back
Divide and Conquer

BIG ASSUMPTION:

THE PROBLEM IS FACTORIZABLE

\[ E_{\text{TOT}} = E_{\text{TKR}} + E_{\text{CAL}} \]

\[ E_{\text{TRK}} = E_{\text{TKR-HITS}} \times F_{\text{TKR-EDGE}} \]

\[ E_{\text{CAL}} = E_{\text{CAL-XTALS}} \times F_{\text{CAL-EDGE}} \times F_{\text{CAL-LEAK}} \]

- \( E_{\text{TKR-HITS}} \) is derived from counting tracker hits
- \( F_{\text{TKR-EDGE}} \) is calculated depending on proximity of Hits to Tower Edges
- \( E_{\text{CAL-XTALS}} \) is derived from CAL Diode Output
- \( F_{\text{CAL-EDGE}} \) is calculated depending on proximity of Hits to Tower Edges
- \( F_{\text{CAL-LEAK}} \) is calculated from the Shower Shape

5 TERMS in ALL!
No. 1: CAL Energy Leakage

Shower Shape model from Wallet Card:

\[
\frac{dE}{d(bt)} = E_0 \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)}
\]

The numerator is just the integrand of the \(\Gamma\) function which on the interval \([0, \infty]\) = \((a-1)! = \Gamma(a)\)

\(b\) is a scale parameter that is \(\sim\) constant with Energy and \(b \sim .5\)

From this the expectation value of \(t\) (Energy Centroid) is

\[
\langle t \rangle = \frac{\int_0^\infty t(bt)^{a-1} e^{-bt} dt}{\int_0^\infty (bt)^{a-1} e^{-bt} dt} = \frac{1}{b} \frac{\Gamma(a+1)}{\Gamma(a)} = \frac{a}{b}
\]

BUT.... We don’t have an infinitely deep Calormeter!
ENERGY Leakage (Part II)

Finite Calorimeter: $\Gamma(a) \rightarrow \Gamma(a, t_{\text{max}}) \text{ (This is the Incomplete } \Gamma \text{ Function)}$ 

As such, we can’t write down in closed form the relationship between $a$ and $\langle \tau \rangle$

But... we can iteratively solve for $a$ given the observed $\langle \tau \rangle$

Specifically:

$$a_{i+1} = a_0 \frac{1 - \frac{\Gamma(a_i, bt_{\text{MAX}})}{\Gamma(a_i)}}{1 - \frac{\Gamma(a_i + 1, bt_{\text{MAX}})}{\Gamma(a_i + 1)}}$$

where $a_0 = \langle t_{\text{OBS}} \rangle b$

And fortunately it converges quickly!

THERE IS NO FITTING!
THE ENERGY CENTROID GIVES THE CORRECTION!
ENERGY Leakage (Part III)

Finally: \[ F_{CAL - LEAK} = \frac{\Gamma(a)}{\Gamma(a, bt_{MAX})} \]

The DEVIL is in the DETAILS!

1) The previously thought 20% gain error in the CAL was wrong. When Leakage is included, energies past ~ 500 MeV are over estimated by numbers approaching 20%. It seems there is a ~ 20 MeV pedestal instead.....

2) You must include the Tracker contributions (t_{TKR}) when computing \( t_{OBS} \)

3) In computing the \( t_{MAX} \) from arclengths, be aware that there is ~ 9mm (front-to-back) of Carbon material giving the calorimeter an effective radiation length of 19.7 mm (CsI is 18.5mm).

4) Obviously \( t_{MAX} \) must include \( t_{TKR} \) as well.

5) The existing formula for the b parameter (see CalRecon) gives much to small values. Presently using \( b = .55 \)
Energy Leakage Results

\[ 1/F_{CAL-LEAK} \]

\[ \frac{E_{RECON} - E_{MC}}{E_{MC}} \]

\[ E_{TKR} \text{ Saturation} \]
No. 2: Tracker Energy

Tracker Energy becomes increasing important at low energies -
Example - 100 MeV $\gamma$'s within 5° of Inst. Axis.

Counts = 12991
Mean = 40.1
Stdev = 15.4

There are 3 pieces to balance:
- Thin Radiator Hits
- Thick Radiator Hits
- Blank Radiator Hits
Tracker Energy – Component Balancing

Issue: How much energy to ascribed per hit?
Depends on Layer due to various radiator thicknesses

Goal: To make energy determination independent of the number of hits in any specific layer

$E_{TKR}/Thin-Hit: .63 \text{ MeV}$

$E_{TKR}/Thick-Hit: 1.67 \text{ MeV}$

Ratio: Thick/Thin = 2.64

Expected 4.3 from radiators

WHY?

In addition: Blank-Hits need .65 MeV/Hit – Probably due to material between TKR & CAL
No. 3: Edge Corrections – The Hardest Part

Basics: Transverse Shower Model
        Circular with a Radially dependent distribution
        Radius given ~ Moliere Radius modulo log(E) dep.

Longitudinal Shower Model
        Cone - saturating to a Cylinder at Shower Max.

Effect of Edges and Gaps: Loss of Observed Energy

Model Correction: Estimate the lost Active Volume
        GLAST is Layers - do estimate layer-by-layer
        Treat Layers as thin-sheets
        Sum of Layers Approximates 3D Integral

Magnitude of the Effect:
        100 MeV ~ 1.5
        10000 MeV ~ 20.

So... ITS BIG and Energy Dependent!
Edge Corrections – Formalism

Shower Profile: Radius = R

Area of a Cord-Defined Slice:

\[ A(r, y) = \pi r^2 \left( \frac{1}{2} - \frac{y}{\pi r} \sqrt{1 - \frac{y^2}{r^2}} - \frac{\sin^{-1} \left( \frac{y}{r} \right)}{\pi} \right) \]

where \( y \) is the distance to the edge (\( y \) goes from 0 to \( r \))
**Edge Corrections – Application**

So the Area of the Active Areas in general will have 2 Pieces:

\[ F_{\text{EDGE}} = \frac{A_{\text{TOWER}_1}(r, y_1) + A_{\text{TOWER}_2}(r, y_2)}{\pi r^2} \]

Given a shower axis, \( y \) can be computed at each plane (in \( z \))
As such a psuedo-3D correction can be computed.

To account for radial dependence - divide into 2 Bins
Core - \( r \) similar to Moliere radius
Fringe - \( r \sim 2 \times \text{Moliere radius} \)

Apply this to the Tracker and to the Calorimeter
(They will have different \( r \)'s and strategies!)

**TRACKER:**
1) Effective radiation length is very large:
   Thin section \( \sim 71 \text{ cm} \) / Thick section \( \sim 16 \text{ cm} \)
2) Start of shower - \( r \) is much smaller the \( r_{\text{Moliere}} \)
   use “fitted value”
3) Plane-to-plane fluctuations in hits make individual
   plane corrections un-workable. Integrate correction
   weighted by plane rad. lens. an apply globally.
Tracker Edge Corrections

100 MeV $\gamma$’s within 5° of Inst. Axis

Parameters:
- gap = 16 mm
- $r_{\text{CORE}}$ = 30 mm
- $r_{\text{FRINGE}}$ = 130 mm
- Frac$_{\text{CORE}}$ = .60

Note: “Over-shoot” near $y = 0$
Its real – more radiation lengths here! (Present correction is too small!)
Calorimeter Edge Corrections

1000 MeV γ's within 5° of Inst. Axis

Parameters: Energy dep. When >1 Track present Dispersion of Tracks at Cal. Entrance Used

Correction is Problematic:
- Observe ~ 150 MeV near the Edge
- Wind up multiplying it by ~ 5
- BUT... there are alot of Events Here! (Within 40 mm of the Edge)

Notice how big the correction has to get!
Results to Date

Conclusions:

1) It is possible to put GLAST Energy back together
2) Tuning the parameters is a slow process due to all the interlocking pieces
3) Off axis - still to be explored!